

## 1 Prime Numbers

A *prime number* is a positive integer other than 1 that is only divisible by 1 and itself.

As you will show in Exercise 1.1, there are infinitely many primes. The number of primes that are smaller than a given natural number  $n$  is denoted  $\pi(n)$ .

### Exercises

*Exercise 1.1* (Euclid's Theorem). Show that there are infinitely many prime numbers.

*Exercise 1.2*. Find an asymptotic formula for  $\pi(n)$ . *Hint*: You might find Exercise 2.1 helpful.

## 2 Zeta function

The zeta function is given by  $\zeta(s) = \sum_{n=1}^{\infty} n^{-s}$ , where  $s$  is a complex number with real part bigger than 1. For example  $\zeta(2) = \frac{\pi^2}{6}$ .

### Exercises

*Exercise 2.1*. Extend  $\zeta$  as far as possible and find all zeros of the function.