

THE DYNKIN DIAGRAMS PACKAGE
VERSION 3.141592

BEN MCKAY

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1. QUICK INTRODUCTION

Load the Dynkin diagram package (see options below)


```
\documentclass{amsart}
\usepackage{dynkin-diagrams}
\begin{document}
The Dynkin diagram of  $(B_3)$  is  $\text{dynkin}\{B\}\{3\}$ .
```

Date: 24 January 2019.

```
\end{document}
```

Invoke it


The Dynkin diagram of (B_3) is `\dynkin{B}{3}`.

The Dynkin diagram of B_3 is .

Inside a *TikZ* statement

The Dynkin diagram of (B_3) is

```
\tikz \dynkin{B}{3};
```

The Dynkin diagram of B_3 is .

Inside a Dynkin diagram environment

The Dynkin diagram of (B_3) is

```
\begin{dynkinDiagram}{B}{3}
```

```
\draw[very thick,red] (root 1) to [out=-45, in=-135] (root 3);
```

```
\end{dynkinDiagram}
```

The Dynkin diagram of B_3 is .

Inside a *TikZ* environment

The baseline controls the vertical alignment:

the Dynkin diagram of (B_3) is

```
\begin{tikzpicture}[baseline=(origin.base)]
```

```
\dynkin{B}{3}
```

```
\draw[very thick,red] (root 1) to [out=-45, in=-135] (root 3);
```

```
\end{tikzpicture}
```

The baseline controls the vertical alignment: the Dynkin diagram of B_3 is



Indefinite rank Dynkin diagrams

```
\dynkin{B}{}
```



Table 1: The Dynkin diagrams of the reduced simple root systems [3] pp. 265–290, plates I–IX

A_n		<code>\dynkin{A}{}</code>
C_n		<code>\dynkin{C}{}</code>
D_n		<code>\dynkin{D}{}</code>
E_6		<code>\dynkin{E}{6}</code>
E_7		<code>\dynkin{E}{7}</code>
E_8		<code>\dynkin{E}{8}</code>
F_4		<code>\dynkin{F}{4}</code>
G_2		<code>\dynkin{G}{2}</code>

2. SET OPTIONS GLOBALLY

Most options set globally ...

```
\pgfkeys{/Dynkin diagram,edge length=.5cm,fold radius=.5cm,
indefinite edge/.style={
  draw=black,fill=white,thin,densely dashed}}
```

You can also pass options to the package in `\usepackage`. *Danger*: spaces in option names are replaced with hyphens: `edge length=1cm` is `edge-length=1cm` as a global option; moreover you should drop the extension `/.style` on any option with spaces in its name (but not otherwise). For example,

...or pass global options to the package

```
\usepackage[
  ordering=Kac,
  edge/.style=blue,
  indefinite-edge={draw=green,fill=white,densely dashed},
  indefinite-edge-ratio=5,
  mark=o,
  root-radius=.06cm]
{dynkin-diagrams}
```

3. COXETER DIAGRAMS




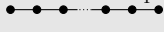

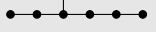
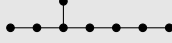





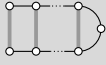
Coxeter diagram option
<code>\dynkin[Coxeter]{F}{4}</code>

gonality option for G_2 and I_n Coxeter diagrams
<code>\(G_2=\dynkin[Coxeter,gonality=n]{G}{2}\), \</code> <code>\(I_n=\dynkin[Coxeter,gonality=n]{I}{n}\)</code>
$G_2 = \overset{n}{\bullet} \bullet$, $I_n = \bullet \overset{n}{\bullet}$

Table 2: The Coxeter diagrams of the simple reflection groups

A_n		<code>\dynkin[Coxeter]{A}{n}</code>
B_n		<code>\dynkin[Coxeter]{B}{n}</code>
C_n		<code>\dynkin[Coxeter]{C}{n}</code>
E_6		<code>\dynkin[Coxeter]{E}{6}</code>
E_7		<code>\dynkin[Coxeter]{E}{7}</code>
E_8		<code>\dynkin[Coxeter]{E}{8}</code>
F_4		<code>\dynkin[Coxeter]{F}{4}</code>
G_2		<code>\dynkin[Coxeter,gonality=n]{G}{2}</code>
H_3		<code>\dynkin[Coxeter]{H}{3}</code>
H_4		<code>\dynkin[Coxeter]{H}{4}</code>
I_n		<code>\dynkin[Coxeter,gonality=n]{I}{n}</code>

4. SATAKE DIAGRAMS

Satake diagrams use the standard name instead of a rank
<code>\(A_{IIIb}=\dynkin{A}{IIIb}\)</code>
$A_{IIIb} =$ 




We use a solid gray bar to denote the folding of a Dynkin diagram, rather than the usual double arrow, since the diagrams turn out simpler and easier to read.

Table 3: The Satake diagrams of the real simple Lie algebras [13] p. 532–534

A_I		<code>\dynkin{A}{I}</code>
A_{II}		<code>\dynkin{A}{II}</code>
A_{IIIa}		<code>\dynkin{A}{IIIa}</code>
A_{IIIb}		<code>\dynkin{A}{IIIb}</code>
A_{IV}		<code>\dynkin{A}{IV}</code>
B_I		<code>\dynkin{B}{I}</code>
B_{II}		<code>\dynkin{B}{II}</code>
C_I		<code>\dynkin{C}{I}</code>
C_{IIa}		<code>\dynkin{C}{IIa}</code>
C_{IIb}		<code>\dynkin{C}{IIb}</code>
D_{Ia}		<code>\dynkin{D}{Ia}</code>
D_{Ib}		<code>\dynkin{D}{Ib}</code>
D_{Ic}		<code>\dynkin{D}{Ic}</code>
D_{II}		<code>\dynkin{D}{II}</code>
D_{IIIa}		<code>\dynkin{D}{IIIa}</code>
D_{IIIb}		<code>\dynkin{D}{IIIb}</code>
E_I		<code>\dynkin{E}{I}</code>
E_{II}		<code>\dynkin{E}{II}</code>
E_{III}		<code>\dynkin{E}{III}</code>
E_{IV}		<code>\dynkin{E}{IV}</code>
E_V		<code>\dynkin{E}{V}</code>
E_{VI}		<code>\dynkin{E}{VI}</code>
E_{VII}		<code>\dynkin{E}{VII}</code>
E_{VIII}		<code>\dynkin{E}{VIII}</code>
E_{IX}		<code>\dynkin{E}{IX}</code>

continued ...

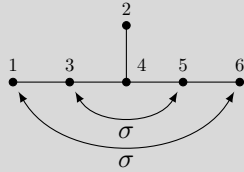
Table 3: ... continued

F_I		<code>\dynkin{F}{I}</code>
F_{II}		<code>\dynkin{F}{II}</code>
G_I		<code>\dynkin{G}{I}</code>

5. HOW TO FOLD

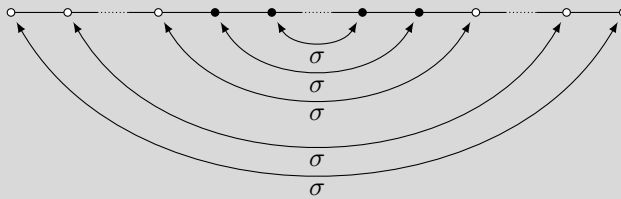
If you don't like the solid gray "folding bar", most people use arrows. Here is E_{II}

```
\newcommand{\invol}[2]{\draw[latex-latex] (root #1) to
[out=-60,in=-120] node[midway,below]{\sigma} (root #2);}
\begin{dynkinDiagram}[edge length=.75cm,labels*={1,...,6}]{E}{6}
\invol{1}{6}\invol{3}{5}
\end{dynkinDiagram}
```



The double arrows for A_{IIIa} are big

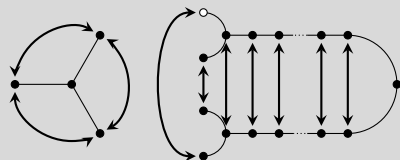
```
\newcommand{\invol}[2]{\draw[latex-latex] (root #1) to
[out=-60,in=-120] node[midway,below]{\sigma} (root #2);}
\begin{dynkinDiagram}[edge length=.75cm]{A}{oo.o**.**o.oo}
\invol{1}{10}\invol{2}{9}\invol{3}{8}\invol{4}{7}\invol{5}{6}
\end{dynkinDiagram}
```



If you don't like the solid gray "folding bar", most people use arrows ...

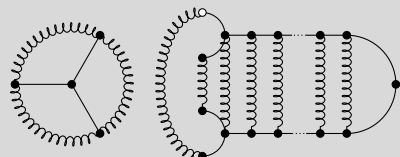
```
\tikzset{/Dynkin diagram/fold style/.style={stealth-stealth,thick,
shorten <=1mm,shorten >=1mm,}}
\dynkin[ply=3,edge length=.75cm]{D}{4}
\begin{dynkinDiagram}[ply=4]{D}{1}%
```

```
{****.****.****}
  \dynkinFold{1}{13}
  \dynkinFold[bend right=90]{0}{14}
\end{dynkinDiagram}
```



...but you could try springs pulling roots together

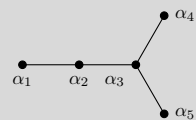
```
\tikzset{/Dynkin diagram/fold style/.style=
{decorate,decoration={name=coil,aspect=0.5,
segment length=1mm,amplitude=.6mm}}}
\dynkin[ply=3,edge length=.75cm]{D}{4}
\begin{dynkinDiagram}[ply=4]{D}[1]%
{****.****.****}
  \dynkinFold{1}{13}
  \dynkinFold[bend right=90]{0}{14}
\end{dynkinDiagram}
```



6. LABELS FOR THE ROOTS

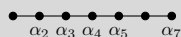
Make a macro to assign labels to roots

```
\dynkin[label,label macro/.code={\alpha_{#1}},edge
length=.75cm]{D}{5}
```



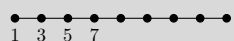
Labelling several roots

```
\dynkin[labels={,2,...,5,,7},label macro/.code={\alpha_{#1}}]{A}{7}
```



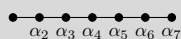
The foreach notation I

```
\dynkin[labels={1,3,...,7},]{A}{9}
```



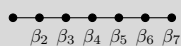
The foreach notation II

```
\dynkin[labels={,\alpha_2,\alpha_...,\alpha_7},]{A}{7}
```



The foreach notation III

```
\dynkin[label macro/.code={\beta_{#1}},labels={,2,...,7},]{A}{7}
```



Label the roots individually by root number

```
\dynkin[label]{B}{3}
```



Label a single root

```
\begin{dynkinDiagram}{B}{3}
\dynkinLabelRoot{2}{\alpha_2}
\end{dynkinDiagram}
```



Use a text style

```
\begin{dynkinDiagram}[text/.style={scale=1.2}]{B}{3};
\dynkinLabelRoot{2}{\alpha_2}
\end{dynkinDiagram}
```



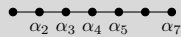
Access root labels via TikZ

```
\begin{dynkinDiagram}{B}{3}
\node[below] at (root 2) {\(\alpha_2\)};
\end{dynkinDiagram}
```



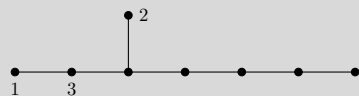
Commands to label several roots

```
\begin{dynkinDiagram}{A}{7}
\dynkinLabelRoots{,\alpha_2,\alpha_3,\alpha_4,\alpha_5,,\alpha_7}
\end{dynkinDiagram}
```



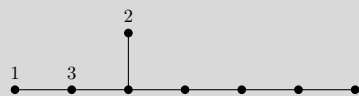
The labels have default locations, mostly below roots

```
\dynkin[edge length=.75cm,labels={1,2,3}]{E}{8}
```



The starred form flips labels to alternate locations, mostly above roots

```
\dynkin[edge length=.75cm,labels*={1,2,3}]{E}{8}
```

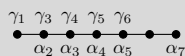


Labelling several roots and alternates

```

\dynkin[%
label macro/.code={\alpha_{#1}},
label macro*/.code={\gamma_{#1}},
labels={,2,...,5,,7},
labels*={1,3,4,5,6}] {A}{7}

```

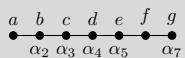


Commands to label several roots

```

\begin{dynkinDiagram}{A}{7}
\dynkinLabelRoots{,\alpha_2,\alpha_3,\alpha_4,\alpha_5,,\alpha_7}
\dynkinLabelRoots*{a,b,c,d,e,f,g}
\end{dynkinDiagram}

```



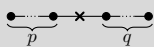
7. BRACING ROOTS

Bracing roots

```

\begin{dynkinDiagram}{A}{*.*x*.*}
\dynkinBrace[p]{1}{2}
\dynkinBrace[q]{4}{5}
\end{dynkinDiagram}

```

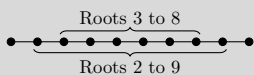


Bracing roots, and a starred form

```

\begin{dynkinDiagram}{A}{10}
\dynkinBrace[\text{Roots 2 to 9}]{2}{9}
\dynkinBrace*[\text{Roots 3 to 8}]{3}{8}
\end{dynkinDiagram}

```



Bracing roots

```

\newcommand\circleRoot[1]{\draw (root #1) circle (3pt);}
\begin{dynkinDiagram}[A]{**.***.***.***.***.***}
\circleRoot{4}\circleRoot{7}\circleRoot{10}\circleRoot{13}
\dynkinBrace[y-1]{1}{3}
\dynkinBrace[z-1]{5}{6}
\dynkinBrace[t-1]{11}{12}
\dynkinBrace[x-1]{14}{16}
\end{dynkinDiagram}

```

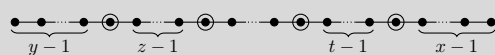
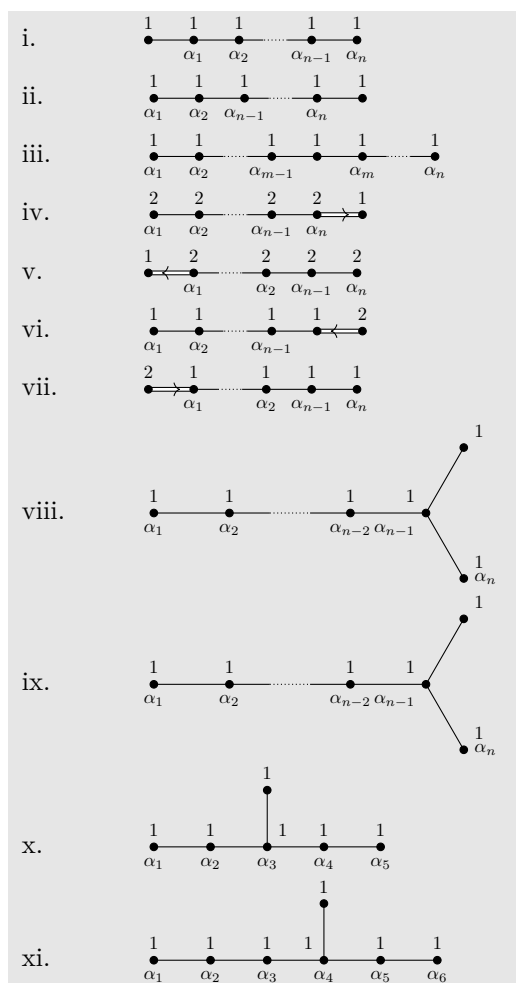
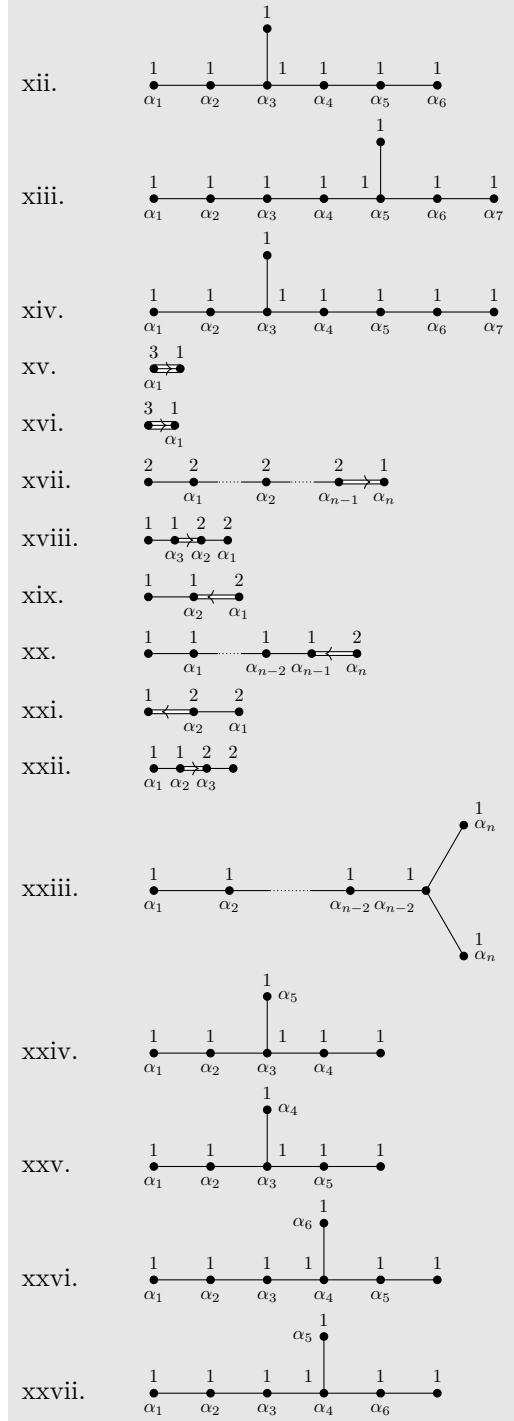


Table 4: Dynkin diagrams from Euler products [17]



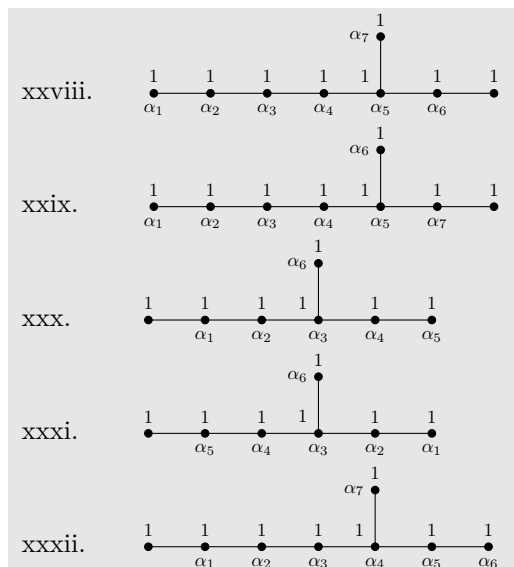
continued ...

Table 4: ... continued



continued ...

Table 4: ... continued



```

\tikzset{/Dynkin diagram,ordering=Dynkin,label macro/.code={\alpha_{#1}}}
\newcounter{EPNo}
\setcounter{EPNo}{0}
\NewDocumentCommand\EP{smmmm}%
{%
\stepcounter{EPNo}\roman{EPNo}. &
\def\eL{.6cm}
\IfStrEqCase{#2}{%
{%
{D}{\gdef\eL{1cm}}%
{E}{\gdef\eL{.75cm}}%
{F}{\gdef\eL{.35cm}}%
{G}{\gdef\eL{.35cm}}%
}%
\tikzset{/Dynkin diagram,edge length=\eL}
\IfBooleanTF{#1}{%
{\dynkin[backwards,labels*={#4},labels={#5}]{#2}{#3}}
{\dynkin[labels*={#4},labels={#5}]{#2}{#3}}
\\
}%
\begin{longtable}{MM}
\caption{Dynkin diagrams from Euler products \cite{Langlands:1967}}\\
\endfirsthead
\caption{\dots continued}\\
\endhead
\multicolumn{2}{c}{continued \dots}\\
\endfoot
\endlastfoot
\EP{A}{***. **}{1,1,1,1,1}{1,2,n-1,n}
\EP{A}{***. **}{1,1,1,1,1}{1,2,n-1,n}
\EP{A}{**.***.*}{1,1,1,1,1,1}{1,2,m-1,,m,n}

```

```

\EP{B}{**.***}{2,2,2,2,1}{1,2,n-1,n}
\EP*{B}{***.**}{2,2,2,2,1}{n,n-1,2,1,}
\EP{C}{**.***}{1,1,1,1,2}{1,2,n-1,}
\EP*{C}{***.**}{1,1,1,1,2}{n,n-1,2,1,}
\EP{D}{**.***}{1,1,1,1,1,1}{1,2,n-2,n-1,n}
\EP{D}{**.***}{1,1,1,1,1,1}{1,2,n-2,n-1,n}
\EP{E}{6}{1,1,1,1,1,1}{1,...,5}
\EP*{E}{7}{1,1,1,1,1,1,1}{6,...,1}
\EP{E}{7}{1,1,1,1,1,1,1}{1,...,6}
\EP*{E}{8}{1,1,1,1,1,1,1,1}{7,...,1}
\EP{E}{8}{1,1,1,1,1,1,1,1}{1,...,7}
\EP{G}{2}{1,3}{,1}
\EP{G}{2}{1,3}{1}
\EP{B}{**.***}{2,2,2,2,1}{,1,2,n-1,n}
\EP{F}{4}{1,1,2,2}{,3,2,1}
\EP{C}{3}{1,1,2}{,2,1}
\EP{C}{**.***}{1,1,1,1,2}{,1,n-2,n-1,n}
\EP*{B}{3}{2,2,1}{1,2}
\EP{F}{4}{1,1,2,2}{1,2,3}
\EP{D}{**.***}{1,1,1,1,1,1}{1,2,n-2,n-2,n,n}
\EP{E}{6}{1,1,1,1,1,1}{1,2,3,4,,5}
\EP{E}{6}{1,1,1,1,1,1}{1,2,3,5,,4}
\EP*{E}{7}{1,1,1,1,1,1,1}{,5,...,1,6}
\EP*{E}{7}{1,1,1,1,1,1,1}{,6,4,3,2,1,5}
\EP*{E}{8}{1,1,1,1,1,1,1,1}{,6,...,1,7}
\EP*{E}{8}{1,1,1,1,1,1,1,1}{,7,5,4,3,2,1,6}
\EP*{E}{7}{1,1,1,1,1,1,1}{5,...,1,,6}
\EP*{E}{7}{1,1,1,1,1,1,1}{1,...,5,,6}
\EP*{E}{8}{1,1,1,1,1,1,1,1}{6,...,1,,7}
\end{longtable}

```

8. STYLE

Colours

```

\dynkin[
  edge/.style={blue!50,thick},
  */.style=blue!50!red,
  arrow color=red]{F}{4}

```



Edge lengths

```

The Dynkin diagram of  $(A_3)$  is \dynkin[edge
  length=1.2,parabolic=3]{A}{3}

```

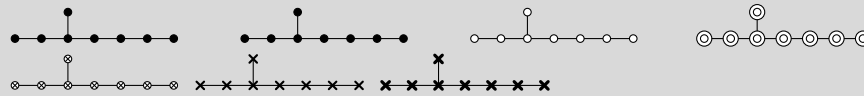
The Dynkin diagram of A_3 is $\times \text{---} \times \text{---} \bullet$

Root marks

```

\dynkin{E}{8}
\dynkin[mark=*]{E}{8}
\dynkin[mark=o]{E}{8}
\dynkin[mark=O]{E}{8}
\dynkin[mark=t]{E}{8}
\dynkin[mark=x]{E}{8}
\dynkin[mark=X]{E}{8}

```

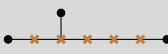


At the moment, you can only use:

- * solid dot
- o hollow circle
- O double hollow circle
- t tensor root
- x crossed root
- X thickly crossed root

Mark styles

The parabolic subgroup $\backslash(E_{8,124})\backslash$ is
`\dynkin[parabolic=124,x/.style={brown,very thick}]{E}{8}`

The parabolic subgroup $E_{8,124}$ is 

Sizes of root marks

$\backslash(A_{3,3})\backslash$ with big root marks is `\dynkin[root radius=.08cm,parabolic=3]{A}{3}`

$A_{3,3}$ with big root marks is 

9. SUPPRESS OR REVERSE ARROWS

Some diagrams have double or triple edges

```
\dynkin{F}{4}
\dynkin{G}{2}
```



Suppress arrows

```
\dynkin[arrows=false]{F}{4}
\dynkin[arrows=false]{G}{2}
```



Reverse arrows

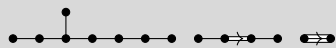
```
\dynkin[reverse arrows]{F}{4}
\dynkin[reverse arrows]{G}{2}
```



10. BACKWARDS AND UPSIDE DOWN

Default

```
\dynkin{E}{8}
\dynkin{F}{4}
\dynkin{G}{2}
```



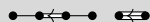
Backwards

```
\dynkin[backwards]{E}{8}
\dynkin[backwards]{F}{4}
\dynkin[backwards]{G}{2}
```



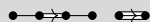
Reverse arrows

```
\dynkin[reverse arrows]{F}{4}
\dynkin[reverse arrows]{G}{2}
```



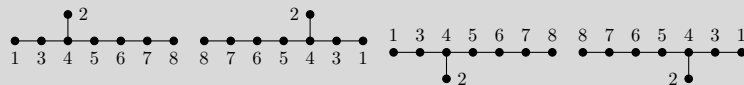
Backwards, reverse arrows

```
\dynkin[backwards,reverse arrows]{F}{4}
\dynkin[backwards,reverse arrows]{G}{2}
```



Backwards versus upside down

```
\dynkin[label]{E}{8}
\dynkin[label,backwards]{E}{8}
\dynkin[label,upside down]{E}{8}
\dynkin[label,backwards,upside down]{E}{8}
```



11. DRAWING ON TOP OF A DYNKIN DIAGRAM

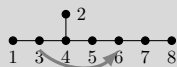
TikZ can access the roots themselves

```
\begin{dynkinDiagram}{A}{4}
  \fill[white,draw=black] (root 2) circle (.15cm);
  \fill[white,draw=black] (root 2) circle (.1cm);
  \draw[black] (root 2) circle (.05cm);
\end{dynkinDiagram}
```



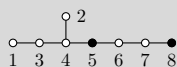
Draw curves between the roots

```
\begin{dynkinDiagram}[label]{E}{8}
  \draw[very thick, black!50,-latex]
    (root 3.south) to [out=-45, in=-135] (root 6.south);
\end{dynkinDiagram}
```



Change marks

```
\begin{dynkinDiagram}[mark=o,label]{E}{8}
  \dynkinRootMark*{5}
  \dynkinRootMark*{8}
\end{dynkinDiagram}
```

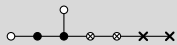


12. MARK LISTS

The package allows a list of root marks instead of a rank:

A mark list

```
\dynkin{E}{oo**ttxx}
```



The mark list `oo**ttxx` has one mark for each root: `o`, `o`, \dots , `x`. Roots are listed in the current default ordering. (Careful: in an affine root system, a mark list will *not* contain a mark for root zero.)

If you need to repeat a mark, you can give a *single digit* positive integer to indicate how many times to repeat it.

A mark list with repetitions

```
\dynkin{A}{x4o3t4}
```

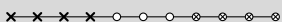


Table 5: Classical Lie superalgebras [10]. We need a slightly larger root radius parameter to distinguish the tensor product symbols from the solid dots.

		<code>\tikzset{/Dynkin diagram,root radius=.07cm}</code>
A_{mn}		<code>\dynkin{A}{o3.oto.oo}</code>
B_{mn}		<code>\dynkin{B}{o3.oto.oo}</code>

continued ...

Table 5: ...continued

B_{0n}		<code>\dynkin{B}{o3.o3.o*}</code>
C_n		<code>\dynkin{C}{too.oto.oo}</code>
D_{mn}		<code>\dynkin{D}{o3.oto.o4}</code>
$D_{21\alpha}$		<code>\dynkin{A}{oto}</code>
F_4		<code>\dynkin{F}{ooot}</code>
G_3		<code>\dynkin[extended,affine mark=t, reverse arrows]{G}{2}</code>

Table 6: Classical Lie superalgebras [10]. Here we see the problem with using the default root radius parameter, which is too small for tensor product symbols.

A_{mn}		<code>\dynkin{A}{o3.oto.oo}</code>
B_{mn}		<code>\dynkin{B}{o3.oto.oo}</code>
B_{0n}		<code>\dynkin{B}{o3.o3.o*}</code>
C_n		<code>\dynkin{C}{too.oto.oo}</code>
D_{mn}		<code>\dynkin{D}{o3.oto.o4}</code>
$D_{21\alpha}$		<code>\dynkin{A}{oto}</code>
F_4		<code>\dynkin{F}{ooot}</code>
G_3		<code>\dynkin[extended,affine mark=t, reverse arrows]{G}{2}</code>

13. INDEFINITE EDGES

An *indefinite edge* is a dashed edge between two roots, $\bullet\text{---}\bullet$ indicating that an indefinite number of roots have been omitted from the Dynkin diagram. In between any two entries in a mark list, place a period to indicate an indefinite edge:

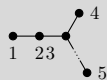
Indefinite edges

`\dynkin{D}{o.o*.*.t.to.t}`

In certain diagrams, roots may have an edge between them even though they are not subsequent in the ordering. For such rare situations, there is an option:

Indefinite edge option

```
\dynkin[make indefinite edge={3-5},label]{D}{5}
```



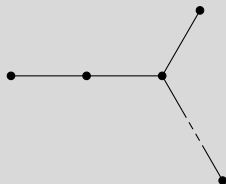
Give a list of edges to become indefinite

```
\dynkin[make indefinite edge/.list={1-2,3-5},label]{D}{5}
```



Indefinite edge style

```
\dynkin[indefinite edge/.style={draw=black,fill=white,thin,densely
dashed},%
edge length=1cm,%
make indefinite edge={3-5}]
{D}{5}
```



The ratio of the lengths of indefinite edges to those of other edges

```
\dynkin[edge length = .5cm,%
indefinite edge ratio=3,%
make indefinite edge={3-5}]
{D}{5}
```

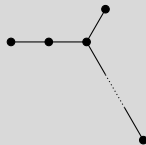


Table 7: Springer’s table of indices [24], pp. 320-321, with one form of E_7 corrected

A_n		
A_n		
B_n		
C_n		
D_n		
E_6		<code>\dynkin{E}{*oooo*}</code>
E_6		<code>\dynkin{E}{o*o*oo}</code>
E_6		<code>\dynkin{E}{o*oooo}</code>
E_6		<code>\dynkin{E}{**ooo*}</code>
E_7		<code>\dynkin{E}{*oooooo}</code>
E_7		<code>\dynkin{E}{ooooo*o}</code>
E_7		<code>\dynkin{E}{oooooo*}</code>
E_7		<code>\dynkin{E}{*oooo*o}</code>
E_7		<code>\dynkin{E}{*oooo**}</code>
E_7		<code>\dynkin{E}{*o**o*o}</code>
E_8		<code>\dynkin{E}{*ooooooo}</code>
E_8		<code>\dynkin{E}{ooooooo*}</code>
E_8		<code>\dynkin{E}{*oooooo*}</code>
E_8		<code>\dynkin{E}{oooooo**}</code>
E_8		<code>\dynkin{E}{*oooo***}</code>
F_4		<code>\dynkin{F}{ooo*}</code>
D_4		<code>\dynkin{D}{o*oo}</code>

14. PARABOLIC SUBGROUPS

Each set of roots is assigned a number, with each binary digit zero or one to say whether the corresponding root is crossed or not:

The flag variety of pointed lines in projective 3-space is associated to the Dynkin diagram $\text{\dynkin[parabolic=3]{A}{3}}$.

The flag variety of pointed lines in projective 3-space is associated to the Dynkin diagram $\times \rightarrow \bullet$.

Table 8: The Hermitian symmetric spaces

A_n		Grassmannian of k -planes in \mathbb{C}^{n+1}
B_n		$(2n - 1)$ -dimensional hyperquadric, i.e. the variety of null lines in \mathbb{C}^{2n+1}
C_n		space of Lagrangian n -planes in \mathbb{C}^{2n}
D_n		$(2n - 2)$ -dimensional hyperquadric, i.e. the variety of null lines in \mathbb{C}^{2n}
D_n		one component of the variety of maximal dimension null subspaces of \mathbb{C}^{2n}
D_n		the other component
E_6		complexified octave projective plane
E_6		its dual plane
E_7		the space of null octave 3-planes in octave 6-space

```

\NewDocumentCommand\HSS{mommm}
{#1&\IfNoValueTF{#2}{\dynkin{#3}{#4}}{\dynkin[parabolic=#2]{#3}{#4}}&#5\}
\renewcommand*\arraystretch{1.5}
\begin{longtable}
{>\columncolor[gray]{.9}>$1<$>\columncolor[gray]{.9}>$1<$>\columncolor[gray]{.9}}1}
\caption{The Hermitian symmetric spaces}\endfirsthead
\caption{\dots continued}\ \endhead
\caption{continued \dots}\ \endfoot
\endlastfoot
\HSS{A_n}{A}{**.*x**}{Grassmannian of $k$-planes in $\mathbb{C}\{n+1\}$}
\HSS{B_n}[1]{B}{-}{$(2n-1)$-dimensional hyperquadric, i.e. the variety of null lines in $\mathbb{C}\{2n+1\}$}
\HSS{C_n}[16]{C}{-}{space of Lagrangian $n$-planes in $\mathbb{C}\{2n\}$}
\HSS{D_n}[1]{D}{-}{$(2n-2)$-dimensional hyperquadric, i.e. the variety of null lines in $\mathbb{C}\{2n\}$}
\HSS{D_n}[32]{D}{-}{one component of the variety of maximal dimension null subspaces of $\mathbb{C}\{2n\}$}
\HSS{D_n}[16]{D}{-}{the other component}
\HSS{E_6}[1]{E}{6}{complexified octave projective plane}
\HSS{E_6}[32]{E}{6}{its dual plane}
\HSS{E_7}[64]{E}{7}{the space of null octave 3-planes in octave 6-space}
\end{longtable}

```

Folded parabolics look bad (zoom in on a root)

```
\dynkin[fold,parabolic=3]{C}{2}
\dynkin[fold,parabolic=3]{G}{2}
```



Folded parabolics: you can try using thicker crosses

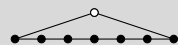
```
\dynkin[fold,x/.style={very thick,line cap=round},parabolic=3]{C}{2}
\dynkin[fold,x/.style={ultra thick,line cap=round},parabolic=3]{G}{2}
```



15. EXTENDED DYNKIN DIAGRAMS

Extended Dynkin diagrams

```
\dynkin[extended]{A}{7}
```



The extended Dynkin diagrams are also described in the notation of Kac [15] p. 55 as affine untwisted Dynkin diagrams: we extend `\dynkin{A}{7}` to become `\dynkin{A}[1]{7}`:

Extended Dynkin diagrams

```
\dynkin{A}[1]{7}
```

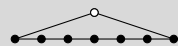


Table 9: The Dynkin diagrams of the extended simple root systems

A_1^1		<code>\dynkin[extended]{A}{1}</code>
A_n^1		<code>\dynkin[extended]{A}{n}</code>
B_n^1		<code>\dynkin[extended]{B}{n}</code>

continued ...

Table 9: ... continued

C_n^1		<code>\dynkin[extended]{C}{}</code>
D_n^1		<code>\dynkin[extended]{D}{}</code>
E_6^1		<code>\dynkin[extended]{E}{6}</code>
E_7^1		<code>\dynkin[extended]{E}{7}</code>
E_8^1		<code>\dynkin[extended]{E}{8}</code>
F_4^1		<code>\dynkin[extended]{F}{4}</code>
G_2^1		<code>\dynkin[extended]{G}{2}</code>

16. AFFINE TWISTED AND UNTWISTED DYNKIN DIAGRAMS

The affine Dynkin diagrams are described in the notation of Kac [15] p. 55:

Affine Dynkin diagrams

`\(A^{(1)}_7=\dynkin{A}[1]{7}, \`
`E^{(2)}_6=\dynkin{E}[2]{6}, \`
`D^{(3)}_4=\dynkin{D}[3]{4}\)`

$A_7^{(1)} =$ $, E_6^{(2)} =$ $, D_4^{(3)} =$

Table 10: The affine Dynkin diagrams

A_1^1		<code>\dynkin{A}[1]{1}</code>
A_n^1		<code>\dynkin{A}[1]{}</code>
B_n^1		<code>\dynkin{B}[1]{}</code>
C_n^1		<code>\dynkin{C}[1]{}</code>
D_n^1		<code>\dynkin{D}[1]{}</code>
E_6^1		<code>\dynkin{E}[1]{6}</code>
E_7^1		<code>\dynkin{E}[1]{7}</code>
E_8^1		<code>\dynkin{E}[1]{8}</code>
F_4^1		<code>\dynkin{F}[1]{4}</code>
G_2^1		<code>\dynkin{G}[1]{2}</code>

continued ...

Table 10: ... continued

A_2^2		<code>\dynkin{A}[2]{2}</code>
A_{ev}^2		<code>\dynkin{A}[2]{even}</code>
A_{od}^2		<code>\dynkin{A}[2]{odd}</code>
D_n^2		<code>\dynkin{D}[2]{}</code>
E_6^2		<code>\dynkin{E}[2]{6}</code>
D_4^3		<code>\dynkin{D}[3]{4}</code>

Table 11: Some more affine Dynkin diagrams

A_4^2		<code>\dynkin{A}[2]{4}</code>
A_5^2		<code>\dynkin{A}[2]{5}</code>
A_6^2		<code>\dynkin{A}[2]{6}</code>
A_7^2		<code>\dynkin{A}[2]{7}</code>
A_8^2		<code>\dynkin{A}[2]{8}</code>
D_3^2		<code>\dynkin{D}[2]{3}</code>
D_4^2		<code>\dynkin{D}[2]{4}</code>
D_5^2		<code>\dynkin{D}[2]{5}</code>
D_6^2		<code>\dynkin{D}[2]{6}</code>
D_7^2		<code>\dynkin{D}[2]{7}</code>
D_8^2		<code>\dynkin{D}[2]{8}</code>
D_4^3		<code>\dynkin{D}[3]{4}</code>
E_6^2		<code>\dynkin{E}[2]{6}</code>

17. EXTENDED COXETER DIAGRAMS

Extended and Coxeter options together

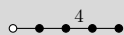
`\dynkin[extended,Coxeter]{F}{4}`

Table 12: The extended (affine) Coxeter diagrams

A_n		<code>\dynkin[extended,Coxeter]{A}{}</code>
B_n		<code>\dynkin[extended,Coxeter]{B}{}</code>
C_n		<code>\dynkin[extended,Coxeter]{C}{}</code>
D_n		<code>\dynkin[extended,Coxeter]{D}{}</code>
E_6		<code>\dynkin[extended,Coxeter]{E}{6}</code>
E_7		<code>\dynkin[extended,Coxeter]{E}{7}</code>
E_8		<code>\dynkin[extended,Coxeter]{E}{8}</code>
F_4		<code>\dynkin[extended,Coxeter]{F}{4}</code>
G_2		<code>\dynkin[extended,Coxeter]{G}{2}</code>
H_3		<code>\dynkin[extended,Coxeter]{H}{3}</code>
H_4		<code>\dynkin[extended,Coxeter]{H}{4}</code>
I_1		<code>\dynkin[extended,Coxeter]{I}{1}</code>

18. KAC STYLE

We include a style called `Kac` which tries to imitate the style of [15].

Kac style

```
\dynkin[Kac]{F}{4}
```

$\circ - \circ \Rightarrow \circ - \circ$

Table 13: The Dynkin diagrams of the simple root systems in Kac style

A_n		<code>\dynkin{A}{}</code>
B_n		<code>\dynkin{B}{}</code>
C_n		<code>\dynkin{C}{}</code>
D_n		<code>\dynkin{D}{}</code>
E_6		<code>\dynkin{E}{6}</code>

continued ...

Table 13: ... continued

E_7		<code>\dynkin{E}{7}</code>
E_8		<code>\dynkin{E}{8}</code>
F_4	$\circ - \circ - \circ \Rightarrow \circ - \circ$	<code>\dynkin{F}{4}</code>
G_2	$\circ \Rightarrow \circ$	<code>\dynkin{G}{2}</code>

Table 14: The Dynkin diagrams of the extended simple root systems in Kac style

A_1^1	$\Leftrightarrow \circ$	<code>\dynkin[extended]{A}{1}</code>
A_n^1		<code>\dynkin[extended]{A}{}</code>
B_n^1		<code>\dynkin[extended]{B}{}</code>
C_n^1	$\circ \Rightarrow \circ - \circ - \dots - \circ - \circ \Leftarrow \circ$	<code>\dynkin[extended]{C}{}</code>
D_n^1		<code>\dynkin[extended]{D}{}</code>
E_6^1		<code>\dynkin[extended]{E}{6}</code>
E_7^1		<code>\dynkin[extended]{E}{7}</code>
E_8^1		<code>\dynkin[extended]{E}{8}</code>
F_4^1	$\circ - \circ - \circ \Rightarrow \circ - \circ$	<code>\dynkin[extended]{F}{4}</code>
G_2^1	$\circ - \circ \Rightarrow \circ$	<code>\dynkin[extended]{G}{2}</code>

Table 15: The Dynkin diagrams of the twisted simple root systems in Kac style

A_2^2	$\circ \Leftarrow \circ$	<code>\dynkin{A}[2]{2}</code>
A_{ev}^2	$\circ \Leftarrow \circ - \circ - \dots - \circ - \circ \Leftarrow \circ$	<code>\dynkin{A}[2]{even}</code>

continued ...

Table 15: ... continued

A_{od}^2		<code>\dynkin{A}[2]{odd}</code>
D_n^2		<code>\dynkin{D}[2]{}</code>
E_6^2		<code>\dynkin{E}[2]{6}</code>
D_4^3		<code>\dynkin{D}[3]{4}</code>

19. CEREF STYLE

We include a style called `ceref` which shapes the root markers more oblongly and with shadows. The word “ceref” is an old form of the word “serif”.

Ceref style	
<code>\dynkin[ceref]{F}{4}</code>	

Table 16: The Dynkin diagrams of the simple root systems in ceref style

A_n		<code>\dynkin{A}{}</code>
B_n		<code>\dynkin{B}{}</code>
C_n		<code>\dynkin{C}{}</code>
D_n		<code>\dynkin{D}{}</code>
E_6		<code>\dynkin{E}{6}</code>
E_7		<code>\dynkin{E}{7}</code>
E_8		<code>\dynkin{E}{8}</code>
F_4		<code>\dynkin{F}{4}</code>
G_2		<code>\dynkin{G}{2}</code>

Table 17: The Dynkin diagrams of the extended simple root systems in ceref style

A_1^1		<code>\dynkin[extended]{A}{1}</code>
---------	--	--------------------------------------

continued ...

Table 17: ... continued

A_n^1		<code>\dynkin[extended]{A}{}</code>
B_n^1		<code>\dynkin[extended]{B}{}</code>
C_n^1		<code>\dynkin[extended]{C}{}</code>
D_n^1		<code>\dynkin[extended]{D}{}</code>
E_6^1		<code>\dynkin[extended]{E}{6}</code>
E_7^1		<code>\dynkin[extended]{E}{7}</code>
E_8^1		<code>\dynkin[extended]{E}{8}</code>
F_4^1		<code>\dynkin[extended]{F}{4}</code>
G_2^1		<code>\dynkin[extended]{G}{2}</code>

Table 18: The Dynkin diagrams of the twisted simple root systems in cref style

A_2^2		<code>\dynkin{A}[2]{2}</code>
A_{ev}^2		<code>\dynkin{A}[2]{even}</code>
A_{od}^2		<code>\dynkin{A}[2]{odd}</code>
D_n^2		<code>\dynkin{D}[2]{}</code>
E_6^2		<code>\dynkin{E}[2]{6}</code>
D_4^3		<code>\dynkin{D}[3]{4}</code>

20. MORE ON FOLDED DYNKIN DIAGRAMS

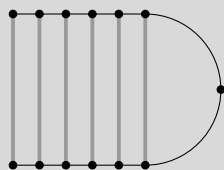
The Dynkin diagrams package has limited support for folding Dynkin diagrams.

Folding

`\dynkin[fold]{A}{13}`

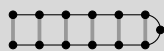
Big fold radius

```
\dynkin[fold, fold radius=1cm]{A}{13}
```



Small fold radius

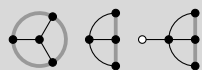
```
\dynkin[fold, fold radius=.2cm]{A}{13}
```



Some Dynkin diagrams have multiple foldings, which we attempt to distinguish (not entirely successfully) by their *ply*: the maximum number of roots folded together. Most diagrams can only allow a 2-ply folding, so `fold` is a synonym for `ply=2`.

3-ply

```
\dynkin[ply=3]{D}{4}
\dynkin[ply=3, fold right]{D}{4}
\dynkin[ply=3]{D}[1]{4}
```



4-ply

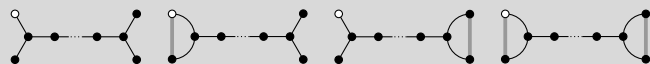
```
\dynkin[ply=4]{D}[1]{4}
```



The $D_\ell^{(1)}$ diagrams can be folded on their left end and separately on their right end:

Left, right and both

```
\dynkin{D}[1]{ } \
\dynkin[fold left]{D}[1]{ } \
\dynkin[fold right]{D}[1]{ } \
\dynkin[fold]{D}[1]{ }
```



We have to be careful about the 4-ply foldings of $D_{2\ell}^{(1)}$, for which we can have two different patterns, so by default, the package only draws as much as it can without distinguishing the two:

Default $D_{2\ell}^{(1)}$ and the two ways to finish it

```
\dynkin[ply=4]{D}[1]{****.*****.*****}%
\
\begin{dynkinDiagram}[ply=4]{D}[1]{****.*****.*****}%
  \dynkinFold[bend right=90]{1}{13}%
  \dynkinFold[bend right=90]{0}{14}%
\end{dynkinDiagram} \
\begin{dynkinDiagram}[ply=4]{D}[1]{****.*****.*****}%
  \dynkinFold{0}{1}%
  \dynkinFold{1}{13}%
  \dynkinFold{13}{14}%
\end{dynkinDiagram}
```

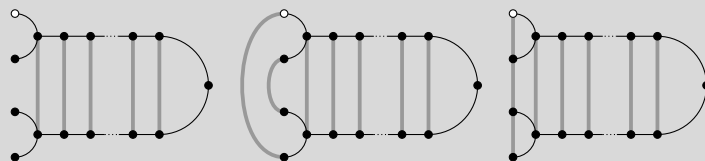


Table 19: Some foldings of Dynkin diagrams. For these diagrams, we want to compare a folding diagram with the diagram that results when we fold it, so it looks best to set `fold radius` and `edge length` to equal lengths.

A_3		<code>\dynkin[fold]{A}[0]{3}</code>
C_2		<code>\dynkin{C}[0]{2}</code>

continued ...

Table 19: ...continued

$A_{2\ell-1}$		<code>\dynkin[fold]{A}{**.****.**}</code>
C_ℓ		<code>\dynkin{C}{}</code>
B_3		<code>\dynkin[fold]{B}{0}{3}</code>
G_2		<code>\dynkin[reverse arrows]{G}{0}{2}</code>
D_4		<code>\dynkin[ply=3, fold right]{D}{4}</code>
G_2		<code>\dynkin{G}{2}</code>
$D_{\ell+1}$		<code>\dynkin[fold]{D}{}</code>
B_ℓ		<code>\dynkin{B}{}</code>
E_6		<code>\dynkin[fold]{E}{0}{6}</code>
F_4		<code>\dynkin[reverse arrows]{F}{0}{4}</code>
A_3^1		<code>\dynkin[ply=4]{A}{1}{3}</code>
A_1^1		<code>\dynkin{A}{1}{1}</code>
$A_{2\ell-1}^1$		<code>\dynkin[fold]{A}{1}{**.****.**}</code>
C_ℓ^1		<code>\dynkin{C}{1}{}</code>
B_3^1		<code>\dynkin[ply=3]{B}{1}{3}</code>
A_2^2		<code>\dynkin{A}{2}{2}</code>
B_3^1		<code>\dynkin[ply=2]{B}{1}{3}</code>
G_2^1		<code>\dynkin{G}{1}{2}</code>
B_ℓ^1		<code>\dynkin[fold]{B}{1}{}</code>
D_ℓ^2		<code>\dynkin{D}{2}{}</code>
D_4^1		<code>\dynkin[ply=3]{D}{1}{4}</code>
B_3^1		<code>\dynkin{B}{1}{3}</code>

continued ...

Table 19: ...continued

D_4^1		<code>\dynkin[ply=3]{D}[1]{4}</code>
G_2^1		<code>\dynkin{G}[1]{2}</code>
$D_{\ell+1}^1$		<code>\dynkin[fold]{D}[1]{\ell}</code>
D_ℓ^2		<code>\dynkin{D}[2]{\ell}</code>
$D_{\ell+1}^1$		<code>\dynkin[fold right]{D}[1]{\ell}</code>
B_ℓ^1		<code>\dynkin{B}[1]{\ell}</code>
$D_{2\ell}^1$		<code>\begin{dynkinDiagram}[ply=4]{D}[1]% {****.*****.*****} \dynkinFold{0}{1} \dynkinFold{1}{13} \dynkinFold{13}{14} \end{dynkinDiagram}</code>
A_{odd}^2		<code>\dynkin{A}[2]{\text{odd}}</code>
$D_{2\ell}^1$		<code>\begin{dynkinDiagram}[ply=4]{D}[1]% {****.*****.*****} \dynkinFold[bend right=90]{1}{13} \dynkinFold[bend right=90]{0}{14} \end{dynkinDiagram}</code>
A_{even}^2		<code>\dynkin{A}[2]{\text{even}}</code>
E_6^1		<code>\dynkin[fold]{E}[1]{6}</code>
F_4^1		<code>\dynkin[reverse arrows]{F}[1]{4}</code>
E_6^1		<code>\dynkin[ply=3]{E}[1]{6}</code>
D_4^3		<code>\dynkin{D}[3]{4}</code>
E_7^1		<code>\dynkin[fold]{E}[1]{7}</code>
E_6^2		<code>\dynkin{E}[2]{6}</code>

continued ...

Table 19: ...continued

F_4^1		<code>\dynkin[fold]{F}[1]{4}</code>
G_2^1		<code>\dynkin{G}[1]{2}</code>
A_{odd}^2		<code>\dynkin[odd, fold]{A}[2]{****.***}</code>
A_{even}^2		<code>\dynkin{A}[2]{even}</code>
D_3^2		<code>\dynkin[fold]{D}[2]{3}</code>
A_2^2		<code>\dynkin{A}[2]{2}</code>

Table 20: Frobenius fixed point subgroups of finite simple groups of Lie type [4] p. 15

$A_{\ell \geq 1}$		<code>\dynkin{A}{}</code>
${}^2A_{\ell \geq 2}$		<code>\dynkin[fold]{A}{}</code>
$B_{\ell \geq 2}$		<code>\dynkin{B}{}</code>
2B_2		<code>\dynkin[fold]{B}{2}</code>
$C_{\ell \geq 3}$		<code>\dynkin{C}{}</code>
$D_{\ell \geq 4}$		<code>\dynkin{D}{}</code>
${}^2D_{\ell \geq 4}$		<code>\dynkin[fold]{D}{}</code>
3D_4		<code>\dynkin[ply=3]{D}{4}</code>
E_6		<code>\dynkin{E}{6}</code>
2E_6		<code>\dynkin[fold]{E}{6}</code>
E_7		<code>\dynkin{E}{7}</code>
E_8		<code>\dynkin{E}{8}</code>
F_4		<code>\dynkin{F}{4}</code>
2F_4		<code>\dynkin[fold]{F}{4}</code>
G_2		<code>\dynkin{G}{2}</code>
2G_2		<code>\dynkin[fold]{G}{2}</code>

21. ROOT ORDERING

Root ordering

```

\dynkin[label,ordering=Adams]{E}{6}
\dynkin[label,ordering=Bourbaki]{E}{6}
\dynkin[label,ordering=Carter]{E}{6}
\dynkin[label,ordering=Dynkin]{E}{6}
\dynkin[label,ordering=Kac]{E}{6}
    
```

Default is Bourbaki. Sources are Adams [1] p. 56–57, Bourbaki [3] p. pp. 265–290 plates I-IX, Carter [5] p. 540–609, Dynkin [8], Kac [15] p. 43.

	Adams	Bourbaki	Carter	Dynkin	Kac
E_6					
E_7					
E_8					
F_4					
G_2					

The marks are set down in order according to the current root ordering:

```

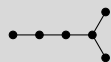
\dynkin[label]{E}{*otxX0t*}
\dynkin[label,ordering=Carter]{E}{*otxX0t*}
\dynkin[label,ordering=Kac]{E}{*otxX0t*}
    
```

22. CONNECTING DYNKIN DIAGRAMS

We can make some sophisticated folded diagrams by drawing multiple diagrams, each with a name:

Name a diagram

```
\dynkin[name=Bob]{D}{6}
```



We can then connect the two with folding edges:

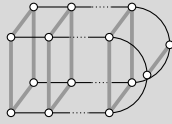
Connect diagrams

```
\begin{dynkinDiagram}[name=upper]{A}{3}
  \node (current) at ($upper root 1)+(0,-.3cm)$ {};
  \dynkin[at=(current),name=lower]{A}{3}
  \begin{scope}[on background layer]
    \foreach \i in {1,...,3}%
    {%
      \draw[/Dynkin diagram/fold style]
        ($upper root \i$)
        -- ($lower root \i$);%
    }%
  \end{scope}
\end{dynkinDiagram}
```



The following diagrams arise in the Satake diagrams of the pseudo-Riemannian symmetric spaces [2].

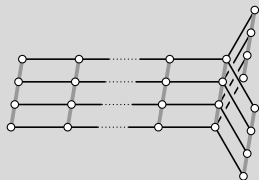
```
\pgfkeys{/Dynkin diagram,edge length=.5cm,fold radius=.5cm}
\begin{tikzpicture}
  \dynkin[name=1]{A}{IIIb}
  \node (a) at (-.3,-.4){};
  \dynkin[name=2,at=(a)]{A}{IIIb}
  \begin{scope}[on background layer]
    \foreach \i in {1,...,7}%
    {%
      \draw[/Dynkin diagram/fold style]
        ($(1 root \i)$)
        --
        ($(2 root \i)$);%
    }%
  \end{scope}
\end{tikzpicture}
```



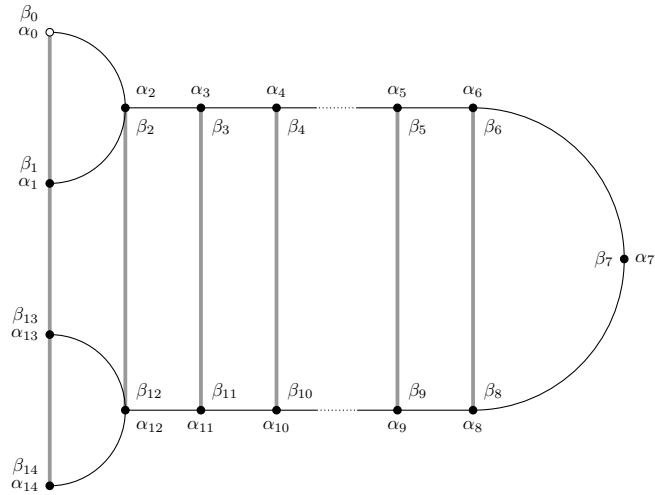
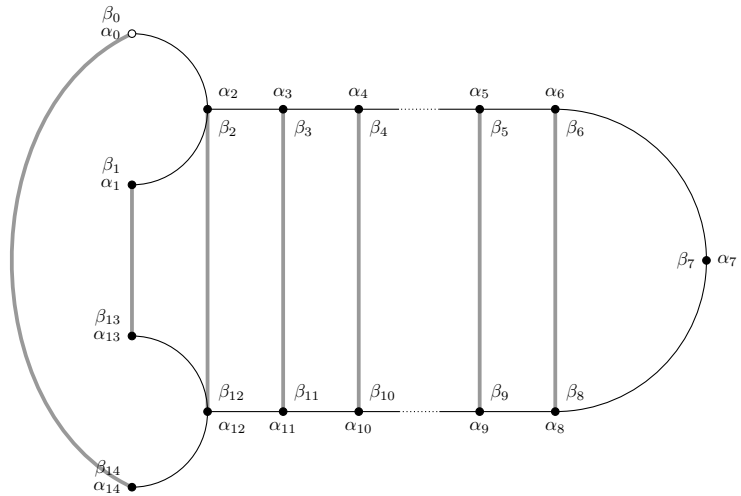
```

\pgfkeys{/Dynkin diagram,
edge length=.75cm,
edge/.style={draw=example-color,double=black,very thick}}
\begin{tikzpicture}
  \foreach \d in {1,...,4}
  {
    \node (current) at ($\d*.05,\d*.3$){};
    \dynkin[name=\d,at=(current)]{D}{oo.oooo}
  }
  \begin{scope}[on background layer]
    \foreach \i in {1,...,6}%
    {%
      \draw[/Dynkin diagram/fold style] ($(1 root
\i)$) -- ($(2 root \i)$);%
      \draw[/Dynkin diagram/fold style] ($(2 root
\i)$) -- ($(3 root \i)$);%
      \draw[/Dynkin diagram/fold style] ($(3 root
\i)$) -- ($(4 root \i)$);%
    }%
  \end{scope}
\end{tikzpicture}

```



23. OTHER EXAMPLES

 1D_4 4-ply tied straight: 1D_4 4-ply tied bending:

```

\tikzset{/Dynkin diagram,edge length=1cm,fold radius=1cm}
\tikzset{/Dynkin diagram,label macro/.code={\alpha_{#1}},label macro*/.code={\beta_{#1}}}
\({}^1 D_4\) 4-ply tied straight:
\begin{dynkinDiagram}[ply=4]{D}[1]%
{****.****.****}
\dynkinFold{0}{1}
\dynkinFold{1}{13}
\dynkinFold{13}{14}
\dynkinLabelRoots{0,...,14}
\dynkinLabelRoots*{0,...,14}
\end{dynkinDiagram}
\({}^1 D_4\) 4-ply tied bending:
\begin{dynkinDiagram}[ply=4]{D}[1]%
{****.****.****}
\dynkinFold{1}{13}
\dynkinFold[bend right=65]{0}{14}
\dynkinLabelRoots{0,...,14}

```

```
\dynkinLabelRoots*{0,...,14}
\end{dynkinDiagram}
```

Below we draw the Vogan diagrams of some affine Lie superalgebras [21, 20].

$\mathfrak{sl}(2m|2n)^{(2)}$

```
\begin{dynkinDiagram}[ply=2,label]{B}[1]{oo.oto.oo}
\dynkinLabelRoot*{7}{1}
\end{dynkinDiagram}
```

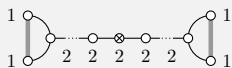
```
\dynkin[label]{B}[1]{oo.oto.oo}
```

```
\dynkin[ply=2,label]{B}[1]{oo.Oto.Oo}
```

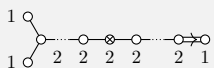
```
\dynkin[label]{B}[1]{oo.Oto.Oo}
```

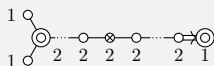
```
\dynkin[label]{D}[1]{oo.oto.ooo}
```

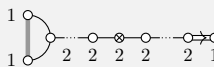
```
\dynkin[label]{D}[1]{o0.ot0.ooo}
```

$$\backslash\text{dynkin}[\text{label}, \text{fold}]\{\text{D}\}[1]\{\text{oo.oto.ooo}\}$$


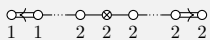
$$\mathfrak{sl}(2m+1|2n)^2$$

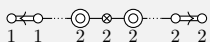
$$\backslash\text{dynkin}[\text{label}]\{\text{B}\}[1]\{\text{oo.oto.oo}\}$$


$$\backslash\text{dynkin}[\text{label}]\{\text{B}\}[1]\{\text{o0.oto.o0}\}$$


$$\backslash\text{dynkin}[\text{label}, \text{fold}]\{\text{B}\}[1]\{\text{oo.oto.oo}\}$$


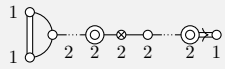
$$\mathfrak{sl}(2m+1|2n+1)^2$$

$$\backslash\text{dynkin}[\text{label}]\{\text{D}\}[2]\{\text{o.oto.oo}\}$$


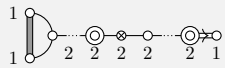
$$\backslash\text{dynkin}[\text{label}]\{\text{D}\}[2]\{\text{o.Ot0.oo}\}$$


$\mathfrak{sl}(2|2n+1)^{(2)}$

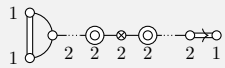
`\dynkin[ply=2,label,double edges]{B}[1]{oo.0to.0o}`



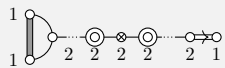
`\dynkin[ply=2,label,double fold]{B}[1]{oo.0to.0o}`



`\dynkin[ply=2,label,double edges]{B}[1]{oo.0t0.oo}`

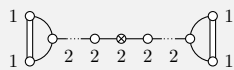


`\dynkin[ply=2,label,double fold]{B}[1]{oo.0t0.oo}`

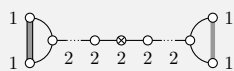


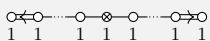
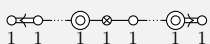
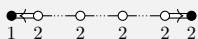
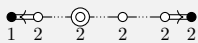
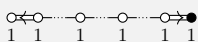
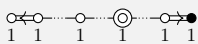
$\mathfrak{sl}(2|2n)^{(2)}$

`\dynkin[ply=2,label,double edges]{D}[1]{oo.oto.ooo}`



`\dynkin[ply=2,label,double fold left]{D}[1]{oo.oto.ooo}`



$osp(2m|2n)^{(2)}$
`\dynkin[label,label macro/.code={1}]{D}[2]{o.o.to.oo}`

`\dynkin[label,label macro/.code={1}]{D}[2]{o.O.to.Oo}`

 $osp(2|2n)^{(2)}$
`\dynkin[label,label macro/.code=\lablIt{#1},
affine mark=*]
{D}[2]{o.o.o.o*}`

`\dynkin[label,label macro/.code=\lablIt{#1},
affine mark=*]
{D}[2]{o.O.o.o*}`

 $sl(1|2n+1)^4$
`\dynkin[label,label macro/.code={1}]{D}[2]{o.o.o.o*}`

`\dynkin[label,label macro/.code={1}]{D}[2]{o.o.O.o*}`


A^1

```

\begin{tikzpicture}
  \dynkin[name=upper]{A}{oo.t.oo}
  \node (Dynkin current) at (upper root 1){};
  \dynkinSouth
  \dynkin[at=(Dynkin
current),name=lower]{A}{oo.t.oo}
  \begin{scope}[on background layer]
    \foreach \i in {1,...,5}{
      \draw[/Dynkin diagram/fold style]
        ($(\upper root \i)$) --
        ($(\lower root \i)$);
    }
  \end{scope}
\end{tikzpicture}

```



```

\dynkin[fold]{A}[1]{oo.t.oooo.t.oo}

```



```

\dynkin[fold,affine mark=t]{A}[1]{oo.o.ootoo.o.oo}

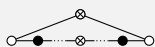
```



```

\dynkin[affine mark=t]{A}[1]{o*.t.*o}

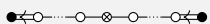
```

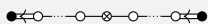
 B^1

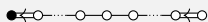
```

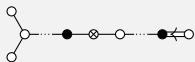
\dynkin[affine mark=*]{A}[2]{o.oto.o*}

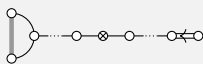
```

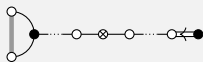


$$\backslash\text{dynkin}[\text{affine mark}=\ast]\{A\}[2]\{o.ot o.o\ast\}$$


$$\backslash\text{dynkin}[\text{affine mark}=\ast]\{A\}[2]\{o.ooo.oo\}$$


$$\backslash\text{dynkin}[\text{odd}]\{A\}[2]\{oo.\ast to.\ast o\}$$


$$\backslash\text{dynkin}[\text{odd, fold}]\{A\}[2]\{oo.ot o.oo\}$$


$$\backslash\text{dynkin}[\text{odd, fold}]\{A\}[2]\{o\ast.ot o.o\ast\}$$

 D^1

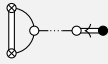
$$\backslash\text{dynkin}\{D\}\{ot oo\}$$


$$\backslash\text{dynkin}\{D\}\{ot \ast o\}$$

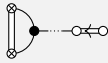

```
\dynkin[fold]{D}{otoo}
```

 C^1

```
\dynkin[double edges, fold, affine  
mark=t, odd]{A}[2]{to.o*}
```



```
\dynkin[double edges, fold, affine  
mark=t, odd]{A}[2]{t*.oo}
```

 F^1

```
\begin{dynkinDiagram}{A}{oto*}%  
  \dynkinQuadrupleEdge{1}{2}%  
  \dynkinTripleEdge{4}{3}%  
\end{dynkinDiagram}%
```



```
\begin{dynkinDiagram}{A}{*too}%  
  \dynkinQuadrupleEdge{1}{2}%  
  \dynkinTripleEdge{4}{3}%  
\end{dynkinDiagram}%
```



G^1

```
\begin{dynkinDiagram}{A}{ot*oo}%
  \dynkinQuadrupleEdge{1}{2}%
  \dynkinDefiniteDoubleEdge{4}{3}%
\end{dynkinDiagram}%
```

```
\begin{dynkinDiagram}{A}{oto*o}%
  \dynkinQuadrupleEdge{1}{2}%
  \dynkinDefiniteDoubleEdge{4}{3}%
\end{dynkinDiagram}%
```

```
\begin{dynkinDiagram}{A}{*too*}%
  \dynkinQuadrupleEdge{1}{2}%
  \dynkinDefiniteDoubleEdge{4}{3}%
\end{dynkinDiagram}%
```

```
\begin{dynkinDiagram}{A}{*tooo}%
  \dynkinQuadrupleEdge{1}{2}%
  \dynkinDefiniteDoubleEdge{4}{3}%
\end{dynkinDiagram}%
```

24. EXAMPLE: THE COMPLEX SIMPLE LIE ALGEBRAS

\mathfrak{g}	Diagram	Weights	Roots	Simple roots
A_n		$\frac{1}{r+1}\mathbb{Z}^{n+1} / \langle \sum e_j \rangle$	$e_i - e_j$	$e_i - e_{i+1}$
B_n		$\frac{1}{2}\mathbb{Z}^n$	$\pm e_i, \pm e_i \pm e_j, i \neq j$	$e_i - e_{i+1}, e_n$
C_n		\mathbb{Z}^n	$\pm 2e_i, \pm e_i \pm e_j, i \neq j$	$e_i - e_{i+1}, 2e_n$
D_n		$\frac{1}{2}\mathbb{Z}^n$	$\pm e_i \pm e_j, i \neq j$	$e_i - e_{i+1}, \quad i \leq n-1$ $e_{n-1} + e_n$

\mathfrak{g}	Diagram	Weights	Roots	Simple roots
E_8		$\frac{1}{2}\mathbb{Z}^8$	$\pm 2e_i \pm 2e_j, \quad i \neq j,$ $\sum_i (-1)^{m_i} e_i, \quad \sum m_i \text{ even}$	$2e_1 - 2e_2,$ $2e_2 - 2e_3,$ $2e_3 - 2e_4,$ $2e_4 - 2e_5,$ $2e_5 - 2e_6,$ $2e_6 + 2e_7,$ $-\sum e_j,$ $2e_6 - 2e_7$
E_7		$\frac{1}{2}\mathbb{Z}^8 / \langle e_1 - e_2 \rangle$	quotient of E_8	quotient of E_8
E_6		$\frac{1}{3}\mathbb{Z}^8 / \langle e_1 - e_2, e_2 - e_3 \rangle$	quotient of E_8	quotient of E_8
F_4		\mathbb{Z}^4	$\pm 2e_i,$ $\pm 2e_i \pm 2e_j, \quad i \neq j,$ $\pm e_1 \pm e_2 \pm e_3 \pm e_4$	$2e_2 - 2e_3,$ $2e_3 - 2e_4,$ $2e_4,$ $e_1 - e_2 - e_3 - e_4$
G_2		$\mathbb{Z}^3 / \langle \sum e_j \rangle$	$\pm(1, -1, 0),$ $\pm(-1, 0, 1),$ $\pm(0, -1, 1),$ $\pm(2, -1, -1),$ $\pm(1, -2, 1),$ $\pm(-1, -1, 2)$	$(-1, 0, 1),$ $(2, -1, -1)$

```

\NewDocumentEnvironment{bunch}{-}{%
{\renewcommand*{\arraystretch}{1}\begin{array}{@{}l@{}}\ \ \midrule{\ \ \midrule\end{array}}
\small
\NewDocumentCommand\nct{mm}{\newcolumnntype{#1}{\columncolor[gray]{.9}}>{\$}m{#2cm}<{\$}}
\nct{G}{.3}\nct{D}{2.1}\nct{W}{3}\nct{R}{3.7}\nct{S}{3}
\NewDocumentCommand\LieG{\mathfrak{g}}
\NewDocumentCommand\W{om}{\ensuremath{\mathbb{Z}}^{#2}\IfValueT{#1}{\left<#1\right>}}
\renewcommand*{\arraystretch}{1.5}
\NewDocumentCommand\quo{\text{quotient of } E_8}
\begin{longtable}{@{}GDWRS@{}}
\LieG&\text{Diagram}&\text{Weights}&\text{Roots}&\text{Simple roots}\ \ \midrule\endfirsthead
\LieG&\text{Diagram}&\text{Weights}&\text{Roots}&\text{Simple roots}\ \ \midrule\endhead
A_n&\dynkin{A}{n}&\frac{1}{n+1}W[\sum e_j]^{n+1}&e_i - e_j, i \neq j &e_i - e_{i+1} \\
B_n&\dynkin{B}{n}&\frac{1}{2}W[n] &\pm e_i, \pm e_i \pm e_j, i \neq j &e_i - e_{i+1}, e_n \\
C_n&\dynkin{C}{n}&\frac{1}{2}W[n] &\pm 2e_i, \pm e_i \pm e_j, i \neq j &e_i - e_{i+1}, 2e_n \\
D_n&\dynkin{D}{n}&\frac{1}{2}W[n] &\pm e_i \pm e_j, i \neq j &e_i - e_{i+1}, e_{n-1} - e_n \\
E_8&\dynkin{E}{8}&\frac{1}{2}W[8] &\pm 2e_i \pm 2e_j, i \neq j, \sum_i (-1)^{m_i} e_i, \sum m_i \text{ even} &2e_1 - 2e_2, 2e_2 - 2e_3, 2e_3 - 2e_4, 2e_4 - 2e_5, 2e_5 - 2e_6, 2e_6 + 2e_7, -\sum e_j, 2e_6 - 2e_7 \\
\end{longtable}
\end{bunch}

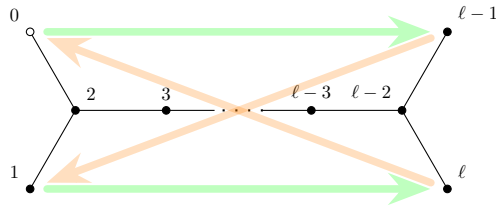
```

```

E_7&\dynkin{E}{7}&\frac{1}{2}\W[e_1-e_2]{8}&\quo&\quo\\
E_6&\dynkin{E}{6}&\frac{1}{3}\W[e_1-e_2,e_2-e_3]{8}&\quo&\quo\\
F_4&\dynkin{F}{4}&\W{4}&
\begin{bunch}\pm 2e_i, \pm 2e_i \pm 2e_j, \quad i \ne j, \pm e_1 \pm e_2 \pm e_3 \pm e_4
\end{bunch}&
\begin{bunch}2e_2-2e_3, 2e_3-2e_4, 2e_4, e_1-e_2-e_3-e_4\end{bunch}\\
G_2&\dynkin{G}{2}&\W[\sum e_j]{3}&
\begin{bunch}
\pm(1,-1,0), \pm(-1,0,1), \pm(0,-1,1), \pm(2,-1,-1), \pm(1,-2,1), \pm(-1,-1,2)
\end{bunch}&
\begin{bunch}(-1,0,1), (2,-1,-1)\end{bunch}
\end{longtable}

```

25. AN EXAMPLE OF MIKHAIL BOROVoi



```

\tikzset{big arrow/.style={
  -Stealth,line cap=round,line width=1mm,
  shorten <=1mm,shorten >=1mm}}
\newcommand\catholic[2]{\draw[big arrow,green!25!white]
(root #1) to (root #2);}
\newcommand\protestant[2]{
\begin{scope}[transparency group, opacity=.25]
\draw[big arrow,orange] (root #1) to (root #2);
\end{scope}}
\begin{dynkinDiagram}[edge length=1.2cm,
indefinite edge/.style={thick,loosely dotted},
labels*={0,1,2,3,\ell-3,\ell-2,\ell-1,\ell}]{D}[1]{
\catholic{0}{6}\catholic{1}{7}
\protestant{7}{0}\protestant{6}{1}
\end{dynkinDiagram}

```


26. SYNTAX

The syntax is `\dynkin[<options>]{<letter>}[<twisted rank>]{<rank>}` where `<letter>` is A, B, C, D, E, F or G, the family of root system for the Dynkin diagram, `<twisted rank>` is 0, 1, 2, 3 (default is 0) representing:

- 0 finite root system
- 1 affine extended root system, i.e. of type ⁽¹⁾
- 2 affine twisted root system of type ⁽²⁾
- 3 affine twisted root system of type ⁽³⁾

and `<rank>` is

- (1) an integer representing the rank or
- (2) blank to represent an indefinite rank or
- (3) the name of a Satake diagram as in section 4.

The environment syntax is `\begin{dynkinDiagram}` followed by the same parameters as `\dynkin`, then various Dynkin diagram and TikZ commands, and then `\end{dynkinDiagram}`.

27. OPTIONS

```

ceref = <true or false>,
default : false
           whether to draw roots in a “ceref” style.
edge length = <number>cm,
default : .35cm
           distance between nodes in the Dynkin diagram
edge/.style = TikZ style data,
default : solid,draw=black,fill=white,thin
           style of edges in the Dynkin diagram
Kac = <true or false>,
default : false
           whether to draw in the style of [15]
name = <string>,
default : anonymous
           A name for the Dynkin diagram, with anonymous treated as a
           blank; see section 22.
parabolic = <integer>,
default : 0
           A parabolic subgroup with specified integer, where the integer
           is computed as  $n = \sum 2^{i-1}a_i$ ,  $a_i = 0$  or 1, to say that root  $i$  is
           crossed, i.e. a noncompact root.
root radius = <number>cm,
default : .05cm
           size of the dots and of the crosses in the Dynkin diagram
text/.style = <TikZ style data>,
default : scale=.7
           Style for any labels on the roots.
mark = <o,0,t,x,X,*>,

```

continued ...

Table 23: ... continued

default : *
 default root mark
affine mark = `o,0,t,x,X,*`,
default : *
 default root mark for root zero in an affine Dynkin diagram
label = `true` or `false`,
default : `false`
 whether to label the roots according to the current labelling scheme.
label macro = `\langle 1-parameter \TeX macro \rangle`,
default : `#1`
 the current labelling scheme for roots.
label macro* = `\langle 1-parameter \TeX macro \rangle`,
default : `#1`
 the current labelling scheme for alternate roots.
make indefinite edge = `\langle edge pair $i-j$ or list of such \rangle`,
default : `{}`
 edge pair or list of edge pairs to treat as having indefinitely many roots on them.
indefinite edge ratio = `\langle float \rangle`,
default : `1.6`
 ratio of indefinite edge lengths to other edge lengths.
indefinite edge/.style = `\langle TikZ style data \rangle`,
default : `solid,draw=black,fill=white,thin,densely dotted`
 style of the dotted or dashed middle third of each indefinite edge.
backwards = `\langle true or false \rangle`,
default : `false`
 whether to reverse right to left.
upside down = `\langle true or false \rangle`,
default : `false`
 whether to reverse up to down.
arrows = `\langle true or false \rangle`,
default : `true`
 whether to draw the arrows that arise along the edges.
reverse arrows = `\langle true or false \rangle`,
default : `true`
 whether to reverse the direction of the arrows that arise along the edges.
fold = `\langle true or false \rangle`,
default : `true`
 whether, when drawing Dynkin diagrams, to draw them 2-ply.
ply = `\langle 0,1,2,3,4 \rangle`,
default : `0`
 how many roots get folded together, at most.
fold left = `\langle true or false \rangle`,
default : `true`
 whether to fold the roots on the left side of a Dynkin diagram.

continued ...

Table 23: ...continued

fold right = $\langle \text{true or false} \rangle$,
default : **true**
 whether to fold the roots on the right side of a Dynkin diagram.

fold radius = $\langle \text{length} \rangle$,
default : **.3cm**
 the radius of circular arcs used in curved edges of folded Dynkin diagrams.

fold style/.style = $\langle \text{TikZ style data} \rangle$,
default : **solid,draw=black!40,fill=none,line width=radius**
 when drawing folded diagrams, style for the fold indicators.

***/.style** = $\langle \text{TikZ style data} \rangle$,
default : **solid,draw=black,fill=black**
 style for roots like \bullet

o/.style = $\langle \text{TikZ style data} \rangle$,
default : **solid,draw=black,fill=black**
 style for roots like \circ

O/.style = $\langle \text{TikZ style data} \rangle$,
default : **solid,draw=black,fill=black**
 style for roots like \odot

t/.style = $\langle \text{TikZ style data} \rangle$,
default : **solid,draw=black,fill=black**
 style for roots like \otimes

x/.style = $\langle \text{TikZ style data} \rangle$,
default : **solid,draw=black,line cap=round**
 style for roots like \times

X/.style = $\langle \text{TikZ style data} \rangle$,
default : **solid,draw=black,thick,line cap=round**
 style for roots like $\mathbf{\times}$

fold left style/.style = $\langle \text{TikZ style data} \rangle$,
default :
 style to override the **fold** style when folding roots together on the left half of a Dynkin diagram

fold right style/.style = $\langle \text{TikZ style data} \rangle$,
default :
 style to override the **fold** style when folding roots together on the right half of a Dynkin diagram

double edges = $\langle \rangle$,
default : **not set**
 set to override the **fold** style when folding roots together in a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows).

double fold = $\langle \rangle$,
default : **not set**

continued ...

Table 23: . . . continued

	set to override the <code>fold</code> style when folding roots together in a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows), but filled in solidly.
<code>double left = $\langle \rangle$,</code>	
<code>default : not set</code>	set to override the <code>fold</code> style when folding roots together at the left side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows).
<code>double fold left = $\langle \rangle$,</code>	
<code>default : not set</code>	set to override the <code>fold</code> style when folding roots together at the left side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows), but filled in solidly.
<code>double right = $\langle \rangle$,</code>	
<code>default : not set</code>	set to override the <code>fold</code> style when folding roots together at the right side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows).
<code>double fold right = $\langle \rangle$,</code>	
<code>default : not set</code>	set to override the <code>fold</code> style when folding roots together at the right side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows), but filled in solidly.
<code>arrow color = $\langle \rangle$,</code>	
<code>default : black</code>	set to override the default color for the arrows in nonsimply laced Dynkin diagrams.
<code>Coxeter = \langletrue or false\rangle,</code>	
<code>default : false</code>	whether to draw a Coxeter diagram, rather than a Dynkin diagram.
<code>ordering = \langleAdams, Bourbaki, Carter, Dynkin, Kac\rangle,</code>	
<code>default : Bourbaki</code>	which ordering of the roots to use in exceptional root systems as in section 21.

All other options are passed to TikZ.

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