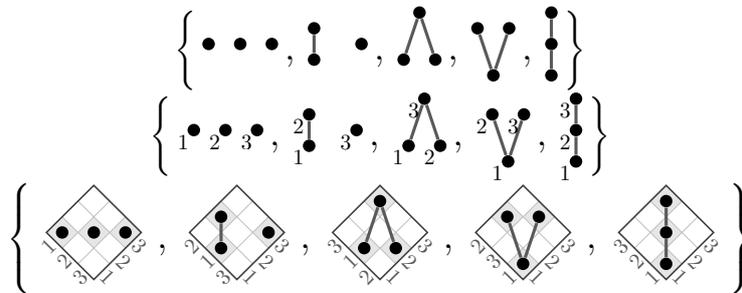package

causets

version 1.4

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What is the purpose of this package?

Causal set theory is a proposed framework for quantum gravity. Causal sets (**causets** for short) are locally finite, partially ordered sets. They are commonly represent with Hasse diagrams, where the partial order (standing for the direction of time) increases towards to the top of the page.

This package implements some simple functionality to produce such diagrams for causets by drawing them as TikZ graphics. The drawing algorithm makes use of the fact that every finite causet that embeds in $1 + 1$ dimensional Minkowski spacetime can be represented by a product of two total orders (2D-orders), where one order can be the total order of the integers labelling the causet elements, while the other is a permutation of the labels[1]. The idea is then used as a blueprint to draw diagrams for any generic (finite) causet, so that it becomes easy to draw causets and even use them in mathematical expressions.

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[1] See https://arxiv.org/abs/0706.0375 and https://arxiv.org/abs/2011.02965 for more details on 2D-orders and causal sets.

1 Introduction - the main commands

This package is built on the package `tikz` to draw the diagrams. To import the `causets` package into a LaTeX file, write

```
1  \usepackage{causets}
```

in the preamble of the document (this will automatically load the `tikz` package as well).

For the command introduction in Table 1, I use the package options `black` (only black and white diagrams) and `permutation` (also shows the permutation),

```
1  \usepackage[black,permutation]{causets}
```

At first, let us take a brief look at the three main commands of the package, before we look into the (physical) ideas behind these three commands.

The three main commands of this package are

```
1  \pcauset{...,i,...,k,...,j,...}
2  \rcauset{...}{...,i/j,...}
3  \causet{...}{...,i/j,...}
```

The first command takes a permutation of the integers from an interval $[a, b]$ (where $N = b - a + 1 \geq 0$ is the cardinality of the causet). The permutation determines the event positions and causal structure such that events i and j are linked if $i < j$ and there is no $k > j$ between i and j in the permutation. The second command does the same, but removes (restyles) links between event pairs `i/j` as given in the second argument. The third command only uses the permutation to position the events, the links have to be explicitly specified by event pairs `i/j` in the second argument.

Any causet that can be embedded in $1 + 1$ dimensional Minkowski spacetime can also be described by the product of two total orders (a 2D-order). Let the causet elements be labeled by an increasing null coordinate u. In the first diagram of Table 1, the u-coordinate axis points upwards on the right hand side. The causal relation (partial order) implies a label permutation along the null coordinate v (pointing upwards along the left axis). This permutation is the input of the `\pcauset` command that computes the causal relation from the permutation. See example 1 in Table 1.

Every causet that does not embed in $1 + 1$ dimensional Minkowski spacetime can be made embeddable by adding links, for which the `\rcauset` command is defined. Find an embedding causet by adding a minimal number of causal relations to the given causet. The so obtained, embedding causet gives a permutation as discussed before for the `\pcauset` causet. The links that have been added to obtain this permutation need to be removed from the generated causet, which is specified by link pairs `i/j` in the second argument. See example 2 in Table 1.

In some situations, it might be easier to specify the list of link pairs that exist, instead of removing links. For this purpose, use the `\causet` command. As an example, we rewrite the example 2 by specifying all existing links as pairs, see example 3 in Table 1. Note that the `\rcauset` command was actually much shorter for this particular example. However,
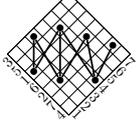
#	command	inline output
1	`\pcauset{4,7,2,6,1,5,3}`	
2	`\rcauset{4,7,2,6,1,5,3}{2/5}`	
3	`\causet{4,7,2,6,1,5,3}{1/3,1/5,2/3,2/6,4/5,4/6,4/7}`	

Table 1: Examples of the three main commands.

if the list of link pairs in the `\rcauset` command is very long, specifying all existing link pairs instead with the `\causet` yields a faster performance.

You may use any of the three main commands to add a causet diagram in-line with text or within a mathematical expression. As an example for the use in a mathematical equation, here is the code for the first set of causets on the title page:

```
\usepackage{causets}
...
\begin{document}
...
\begin{align}
  S &=
  \left\{
    \pcauset{3,2,1}, \pcauset{3,1,2}, \pcauset{2,1,3}, \pcauset{1,3,2},
    \pcauset{1,2,3}
  \right\}.
\end{align}
```

$$S = \left\{ \cdots, \vdots \cdot, \wedge, \vee, \vdots \right\}. \tag{1}$$

Of course, a given causet might have more than one representing permutation, so that any of those permutations may be used. However, you may consider it a good practice to choose the permutation that starts with the larger integer, so that disjoint elements appear to the right in the graphs. For example, you may write `\pcauset{3,1,2}`, $\vdots \cdot$, as opposed to `\pcauset{2,3,1}`, $\cdot \vdots$.

The arguments of the commands are lists that are iterated over by a `\foreach` loop of PGF/TikZ. So you may use dots ... to complete ranges of integers automatically. You may use this feature to draw chains like $\vdots$ (with `\pcauset{1,...,5}`) or antichains like $\cdots\cdots$ (with `\pcauset{5,...,1}`). As a more complex example for the dots notation, consider
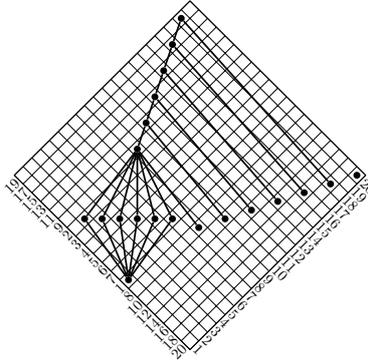
```
\pcauset{20,18,...,8, 1,7,6,...,2, 9,11,...,19}
```

which produces the same diagram as

```
1 \pcauset{20,18,16,14,12,10,8, 1,7,6,5,4,3,2, 9,11,13,15,17,19}
```


In an optional argument, the main commands accepts any causets or TikZ keys to change the style of the causet. For example, the blue causet is obtained with

```
1 \pcauset[blue colors]{2,4,5,1,3}
```

2   Package options

Each of the package options can either be specified in square brackets when declaring the package (option applies globally) or be changed with a macro at any point in the document (option applies to the local scope).
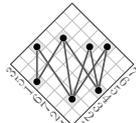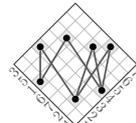
If no options are used, an output like is obtained with the code

```
1 \usepackage{causets}
2 ... \begin{document} ...
3 \rcauset{4,2,7,6,1,5,3}{2/5}
```

The options – given in the following table – are passed to the causets package as optional parameter and can be combined. The package options are listed together with example outputs created with the commands
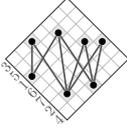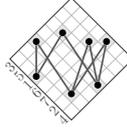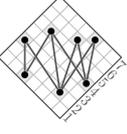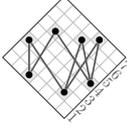
```
1 \pcauset{4,2,7,6,1,5,3}
2 \rcauset{4,2,7,6,1,5,3}{2/5}
```

If not specified otherwise, any package option also works with the command \causet.

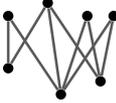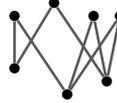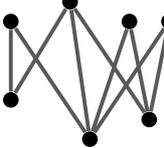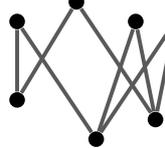| Package option	\pcauset	\rcauset
\usepackage[permutation]{causets} shows permutations (\causetsDrawPermutationtrue activates this option) (\causetsDrawPermutationfalse deactivates this option)		

<center>continued on the next page</center>

Package option	\pcauset	\rcauset
`\usepackage[permutation, uunlabeled]{causets}` shows permutations without u-labels (`\causetsDrawULabelstrue` activates u-labels) (`\causetsDrawULabelsfalse` deactivates u-labels)		
`\usepackage[permutation, vunlabeled]{causets}` shows permutations without v-labels (`\causetsDrawVLabelstrue` activates v-labels) (`\causetsDrawVLabelsfalse` deactivates v-labels)		
`\usepackage[labeled]{causets}` shows labeled events (`\causetsDrawLabelstrue` activates labels) (`\causetsDrawLabelsfalse` deactivates labels)		
`\usepackage[continuous]{causets}` allows links to cross over events. Link interruptions (default) only affect the `\causet` command and "restyled" links of `\rcauset`, but not the examples of shown on the right. Details on interrupted (broken) links are given below this table.		
`\usepackage[caps]{causets}` sets caps as link tips		
`\usepackage[arrows]{causets}` sets arrows (rounded stealth) as link tips		
`\usepackage[unlinked]{causets}` does not draw links (`\causetsDrawLinkstrue` actives links) (`\causetsDrawLinksfalse` deactivates links)		
`\usepackage[tiny]{causets}` for tiny diagrams (`\tikzcausetsset{tiny}` sets tiny sizes)		
`\usepackage[smaller]{causets}` for very small diagrams (`\tikzcausetsset{very small}` sets very small sizes)		
`\usepackage[small]{causets}` for small diagrams (`\tikzcausetsset{small}` sets small sizes)		

Package option	\pcauset	\rcauset
For comparison, here are the normal sized causets. (\tikzcausetsset{normal} sets normal sizes)		
\usepackage[large]{causets} for large diagrams (\tikzcausetsset{large} sets large sizes)		
\usepackage[larger]{causets} for very large diagrams (\tikzcausetsset{very large} sets very large sizes)		
\usepackage[huge]{causets} for huge diagrams (\tikzcausetsset{huge} sets huge sizes) Details on size options are given in Sec. 5.		
\usepackage[black]{causets} for black and white causets (\tikzcausetsset{black colors} activates the option)		
\usepackage[gray]{causets} for grey scale causets (default) (\tikzcausetsset{gray colors} activates the option)		
\usepackage[blue]{causets} for blue causets (\tikzcausetsset{blue colors} activates the option)		
\usepackage[neon]{causets} for brightly coloured causets (\tikzcausetsset{neon colors} activates the option) Details on colour options are given in Sec. 6.		
\usepackage[spatial]{causets} shows dashed lines between spatially separated events (which is a secondary partial order for the causets drawn with \pcauset). This option only affects the \pcauset and \rcauset commands, but not the \causet command. (\causetsDrawSpatialLinkstrue activates the option) (\causetsDrawSpatialLinksfalse deactivates the option)		
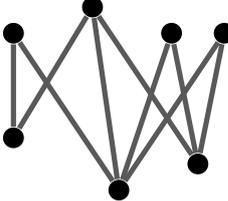
| *continued from the previous page*		
Package option	\pcauset	\rcauset
\usepackage[external]{causets} loads the external library, sets the external file name prefix to "causets/", and enables the use of causet names for the external file names. (This feature has to be loaded as option in the preamble not later in the document.) For details on externalization of causets, see Sec. 8.		

By default, the links in a diagram are interrupted if they cross over another event (exactly over the event centre). Any such crossing of links over events can only occur for the links explicitly specified for the \causet command or the locally restyled links of the \rcauset command, but not for the links drawn by \pcauset or automatically drawn links by \rcauset. This feature is activated by default (\causetsBreakLinkstrue) and can be deactivated with \causetsBreakLinksfalse, or with the option continuously linked.

For example, here is a causet that embeds in 3-dimensional but not in 2-dimensional Minkowski spacetime. This causet has one event (0) in the centre that is linked to events -3 and 3 but not to events -6 and 6, while a link (blue) from -6 to 6 crosses over it.

```
1 \usepackage[larger]{causets}
2 \tikzcausetsset{labels={right, position=east}}
3 ... \begin{document} ...
4 \causet[brokenly linked]{-1,2,-5,-7,4,-3,-6,0,6,3,-4,7,5,-2,1}
    {-7/-6,-7/4,-6/-4,-6/6/blue!50!cyan,-5/-4,-5/-3,-4/-2,-4/7,
    -3/-2,-3/0,-2/1,-1/0,-1/2,0/1,0/3,2/3,2/4,3/5,4/5,4/6,6/7}
5 \causet[continuously linked]{-1,2,-5,-7,4,-3,-6,0,6,3,-4,7,5,-2,1}
    {-7/-6,-7/4,-6/-4,-6/6/blue!50!cyan,-5/-4,-5/-3,-4/-2,-4/7,
    -3/-2,-3/0,-2/1,-1/0,-1/2,0/1,0/3,2/3,2/4,3/5,4/5,4/6,6/7}
```
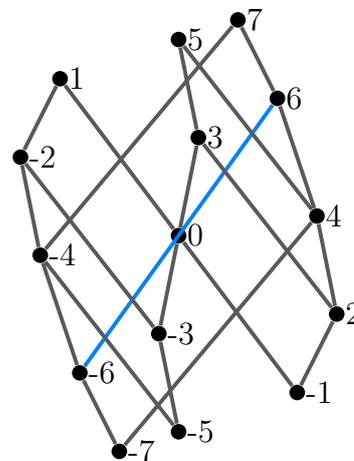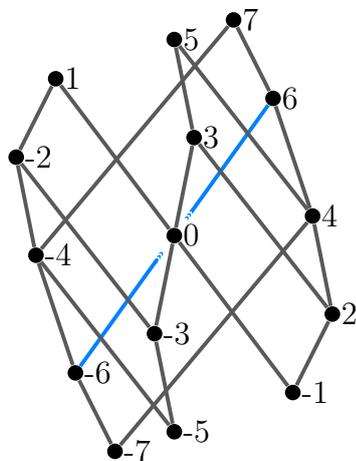
Note that the links are drawn in the order given by the second argument. Similar results are obtained with the following \rcauset commands (since version 1.4)

```
1 \rcauset[brokenly linked]{-1,2,-5,-7,4,-3,-6,0,6,3,-4,7,5,-2,1}
    {-7/-3,-6/0,-5/4,-4/5,0/6,3/7,-6/6/blue!50!cyan}
2 \rcauset[continuously linked]{-1,2,-5,-7,4,-3,-6,0,6,3,-4,7,5,-2,1}
    {-7/-3,-6/0,-5/4,-4/5,0/6,3/7,-6/6/blue!50!cyan}
```

where all pairs of integers in the second argument are links to be removed (by default, get the local style `draw=none`). All links with a local style specified (here the last link only) are drawn in the order they appear after all automatic links have been drawn. Local styles are explained in Sec. 7 in more detail.

All main commands, the short hand macros (see Sec. 3), and the command `\causetfile` (see Sec. 8) add a small padding to the left and right of the diagram for a better spacing within text and mathematical expressions. The padding is adjustable with `padding=⟨value⟩` and stored in the macro `\causets@Padding` (by default, it is `\thinspace`).

3 Short hand macros

The permutations and labels for causets can be switched on globally with the package options `permutation` and `labeled`, or at any point in the document with the switches `\causetsDrawPermutationtrue` (same as the command `\tikzcausetsset{show permutation}`) and `\causetsDrawLabelstrue` (or `\tikzcausetsset{labeled}`), respectively.

To show only single causets with either of these options, there exist short hand macros for all three main commands. Each main command can be followed by a capital letter `P` (to show the permutation only), `L` (to show event labels only), or `X` (to show both).

```
1 Causets with permutations (and labels) \pcausetP{4,1,2,3},
    \pcausetX{1,2,3,4}, \pcausetL{1,2,4,3}, and without any
    \pcauset{2,4,1,3}.
```

Causets with permutations (and labels) , , , and without any .

These results are also obtained by using the optional parameter, for example:

```
1 Causets with permutations (and labels) \pcauset[permutation]{4,1,2,3},
    \pcauset[permutation,labeled]{1,2,3,4}, \pcauset[labeled]{1,2,4,3},
    and without any \pcauset{2,4,1,3}.
```

There are a few short hand macros for small common causets as shown here.

```
1 \causetFence{1}, \causetFence{2}, \ldots, \causetFence{10},
2 \causetCrown{1}, \causetCrown{2}, \ldots, \causetCrown{10}
```


4 Labeling and replacing events (also to represent infinite causets)

Since version v1.3 of the package, it is possible to relabel and replace causet events. These features are useful for labelling events arbitrarily, and drawing diagrams of infinite causets using ellipsis.

By default, all events get named and labeled by the integers given in the arguments of the commands. For example, `\pcausetL{0,2,1}` creates the diagram ¹⸾Ⅴ̷ where the event notes are named (E0), (E2), (E1), respectively, while the label nodes have the names (EL0), (EL2), (EL1). If the permutation is displayed as well, the *u*- and *v*-coordinate labels are the nodes (EUL⟨*value*⟩) and (EVL⟨*value*⟩), with ⟨*value*⟩ taking on the values {0, 2, 1}.

To offset the names of all causet events by an integer amount, use the key-value pair `offset=`⟨*value*⟩ to adjust the naming and labelling of events and get distinct event names, while the same integers are used in the command arguments. Consider the following example.

```
1  \begin{align*}
2    \tikzcausetsset{large}
3    \cdots\quad
4    \pcausetP[offset=-12]{0,2,1,3}\quad\cdots\quad
5    \pcausetL{0,2,1,3}\quad\cdots\quad
6    \pcausetL[offset=12]{0,2,1,3}\quad\cdots
7  \end{align*}
```
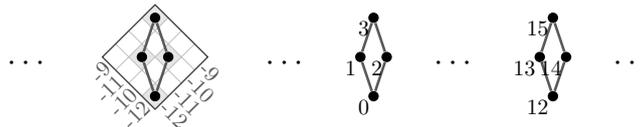

Here, the three commands have the same integer argument {0,2,1,3}, but the events are named with the specified offsets for the sub-causets, and the labels are adjusted accordingly. In this way, it will be possible to address the event (and label) nodes of the sub-causets distinctively within a larger TikZ picture. For more details on how to draw causets within a TikZ picture, see Sec. 10.

When you (only) want to change the labelling of an event to ⟨*value*⟩, add a slash and `label=`⟨*value*⟩ directly after the event integer in the first command argument. For example, the causet ⋀ has six different labellings by (distinct) integers from the set {0, 1, 2},

```
1   \begin{align}
2     \text{order-preserving:} &&
3     L_{\mathrm{op}} &=
4     \left\{
5       \pcausetL{1,0,2},
6       \pcausetL{1/label=0,0/label=1,2}
7     \right\}
8     , \\
9     \text{not order-preserving:} &&
10    L_{\mathrm{nop}} &=
11    \left\{
12      \pcauset{1/label=2,0/label=1,2/label=0},
13      \pcausetL{1,0/label=2,2/label=0},
14      \pcausetL{1/label=2,0,2/label=1},
15      \pcauset{1/label=0,0/label=2,2/label=1}
16    \right\}
```
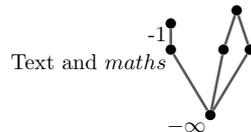
```
17    .
18 \end{align}
```

$$\text{order-preserving:} \qquad L_{\text{op}} = \left\{ {}_{0}\!\!\bigwedge_{1}^{2}\bullet, {}_{1}\!\!\bigwedge_{0}^{2}\bullet \right\}, \tag{2}$$

$$\text{not order-preserving:} \qquad L_{\text{nop}} = \left\{ {}_{1}\!\!\bigwedge_{2}^{0}, {}_{2}\!\!\bigwedge_{1}^{0}, {}_{0}\!\!\bigwedge_{2}^{1}, {}_{2}\!\!\bigwedge_{0}^{1} \right\}. \tag{3}$$

The replacing labels can be any text. Longer text labels should be written in a TEX group {...}. Here is an example

```
1 \pcauset[large]{2/label=$-\infty$,6,5,7,3/label={Text and
    $maths$},4/label=-1}
```


Notice that replaced labels will always be visible, even if labels are deactivated (\causetsDrawLabelsfalse) as in this example. To change the style of the replaced labels, add the key-value pair replaced labels=⟨value⟩, which extends the style of standard labels.

Similarly to the replacement of labels, you can replace event nodes by using a slash followed by event=⟨value⟩. This feature may have its main application for the representation of infinite causets, for example, the code

```
1 % Define ellipsis rotated by #1 degrees:
2 \newcommand{\rdots}[1][0]{\rotatebox{#1}{...}}
3 ...
4 % Draw infinite causet with ellipsis
5 \pcauset[large, replaced labels={below, at=south}, replaced
    events={text scale=1}]
6  {-2/{label={$\downarrow$ continues infinitely $\downarrow$},
    event={\rdots[90]}}, -1,0,1,16,14,15,11,12,13,7,8,9,10,
    2/event={\rdots[10]},3,4/event={~},5,6/event={\rdots[-10]}}
```

yields the diagram
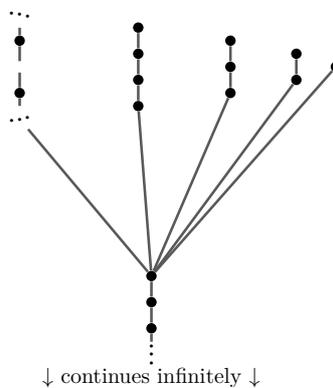
Here, three events are replaced by ellipsis with different rotations, and one event is replaced by an empty node (~). The style of the replaced events is specified via the key replaced labels=⟨value⟩ – where ⟨value⟩ is {text scale=1} in this example. This example also shows how to relabel and replace simultaneously (see the bottom-most event).

10

The style and the position of all event labels is set with `labels=`⟨*value*⟩, while `replaced labels=`⟨*value*⟩ extends this style. As shown in the previous example, the position of labels is defined with a key word like `left`, `right`, `above`, `below`, etc. (see also the TikZ manual) relative to the position given by the key `at=`⟨*value*⟩. The `at` key holds a coordinate of the event note. The default positioning is `below left, at=south west`.

5   Changing sizes and fonts (for mathematical expressions)

The pre-defined size styles have the following scaling with respect to the normal size:

- `tiny` is $\frac{1}{3}$ times the normal size
- `very small` is $\frac{4}{9}$ times the normal size
- `small` is $\frac{2}{3}$ times the normal size
- `large` is $\frac{3}{2}$ times the normal size
- `very large` is $\frac{9}{4}$ times the normal size
- `huge` is 3 times the normal size

Apart from the pre-defined sizes, a causet can be scaled to any size using `size unit=`⟨*value*⟩ (which is equal to the tile size, `tile size=`⟨*value*⟩). The normal size unit has the value `0.9ex`.

I recommend to change the size of a causet when used in indices or limits for mathematical expression. For a (first level) sub-script in a summation, you may use the small size,

```
1  \begin{equation}
2      \sum_{C \in \tikzcausetsset{small}
3        \Bigl\{ \pcauset{1,2,3}, \pcauset{1,3,2,4}, \pcauset{1,4,3,2,5},
     \pcauset{1,5,4,3,2,6} \Bigr\}} | C |
4   = \left| \pcauset{1,2,3} \right|
5      + \left| \pcauset{1,3,2,4} \right|
6      + \left| \pcauset{1,4,3,2,5} \right|
7      + \left| \pcauset{1,5,4,3,2,6} \right|
8  \end{equation}
```

$$\sum_{C \in \left\{ \vcenter{}, \vcenter{}, \vcenter{}, \vcenter{} \right\}} |C| = \left| \vcenter{} \right| + \left| \vcenter{} \right| + \left| \vcenter{} \right| + \left| \vcenter{} \right| \tag{4}$$

Like any other options, sizes can also be specified in the optional argument of a single causet diagram or with `\tikzcausetsset{...}` for the current scope.

If you need to set the sizes of individual components of a causet, use the keys

- `tile size=`⟨*value*⟩ to store ⟨*value*⟩ in the macro `\causetTileSize` as the size of the tiles in the permutation (this is equivalent to setting the size unit),
- `region line width=`⟨*value*⟩ to set the thickness for the line surrounding the permutation region,
- `grid line width=`⟨*value*⟩ to set the line thickness of the permutation grid,
- `event size=`⟨*value*⟩ to set the diameter of the events,

- `event outline=⟨value⟩` to set the thickness of the transparent line around the events,
- `link width=⟨value⟩` to set the link thickness, and
- `broken link gap=⟨value⟩` to set the link gap before and after an event that interrupts a link.

The text of all labels is sized independently to the other components of the causet diagrams. To adjust the text typesetting, use

- `text font=⟨value⟩` to set the font used for all the text labels – for example, `text font=\fontsize{13}{15.6}\selectfont` sets the font size to 13pt (and the baseline skip to 1.2 times this font size),
- `text scale=⟨value⟩` to set the scaling factor of all the text labels (relative to the font size).

The pre-defined size styles use the following font sizes,

- `tiny` is typeset with `\tiny` (scaled by 0.30)
- `very small` is typeset with `\tiny` (scaled by 0.40)
- `small` is typeset with `\scriptsize` (scaled by 0.43)
- `normal` is typeset with `\footnotesize` (scaled by 0.56)
- `large` is typeset with `\small` (scaled by 0.75)
- `very large` is typeset with `\normalsize` (scaled by 1.02)
- `huge` is typeset with `\Large` (scaled by 0.97)

Below is an example for a diagram drawn with some sizes defined manually:

```
\pcausetL[tile size=2ex, event size=1ex, link width=0.3ex, text
    scale=1]{13,8,14,9,3,15,12,1,6,16,5,11,7,2,10,4}
```
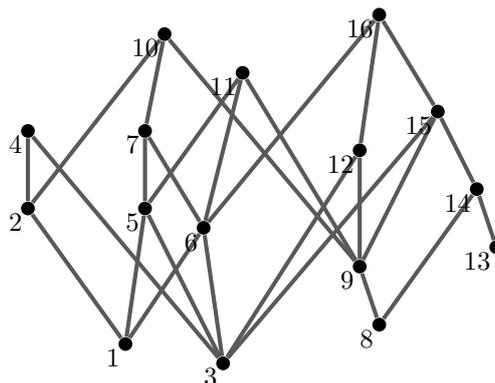

6 Changing colours

Colors can be set with the following keys:

- `region color=⟨value⟩` (or `region color=⟨value⟩`) for the frame of the permutation grid
- `grid color=⟨value⟩` (or `grid color=⟨value⟩`) for the lines of the permutation grid
- `tile color=⟨value⟩` (or `tile color=⟨value⟩`) for the occupied tiles

- `ulabel color=`⟨*value*⟩ (or `ulabel color=`⟨*value*⟩) for the u labels
- `vlabel color=`⟨*value*⟩ (or `vlabel color=`⟨*value*⟩) for the v labels
- `event color=`⟨*value*⟩ (or `event color=`⟨*value*⟩) for the events
- `label color=`⟨*value*⟩ (or `label color=`⟨*value*⟩) for the event labels
- `link color=`⟨*value*⟩ (or `link color=`⟨*value*⟩) for the links
- `spatial link color=`⟨*value*⟩ (or `spatial link color=`⟨*value*⟩) for the spatial links.

When using any of these key-value pairs in an optional arguments of the causet macros, they apply to that diagram. To change colours within a scope or the entire document, use the `\tikzcausetsset` macro, for example

```
1  \tikzcausetsset{event color=purple, link color=cyan}
```

changes the event colour to purple and the link colour to cyan.

For the pre-defined colour schemes, use the keys `gray colors` (or `gray colors`, default), `black colors` (or `black colors`, only black and white), `blue colors` (or `blue colors`, blue colour tones), or `neon colors` (or `neon colors`, bright colours). These keys set all the colours listed above to pre-defined values.

7 Changing the styles of individual events and links

The full supported formats of the three main commands are

`\pcauset[`⟨*causet style*⟩`]{`⟨*index*⟩`/`⟨*event style*⟩`,...}`

`\rcauset[`⟨*causet style*⟩`]{`⟨*index*⟩`/`⟨*event style*⟩`,...}{`⟨*unlink from*⟩`/`⟨*unlink to*⟩`,...,` ⟨*link from*⟩`/`⟨*link to*⟩`/`⟨*link style*⟩`,...}`

`\causet[`⟨*causet style*⟩`]{`⟨*index*⟩`/`⟨*event style*⟩`,...}{`⟨*link from*⟩`/`⟨*link to*⟩`/`⟨*link style*⟩`,...}`

and similarly for all short hand macros as described in Sec. 3. The style values are all optional, where ⟨*causet style*⟩ is applied to that causet only, ⟨*event style*⟩ is applied to a single event, and ⟨*link style*⟩ is applied to a single link.

The feature to apply styles only to individual events and links is to highlight or modify subsets of a causet, for example, by using different colours:

```
1  \rcausetL[large, subset A/.style={red, dotted}, subset B/.style=blue,
      replaced labels={text scale=1, at=east, right}]{2/{subset A,
      label=event $a$}, 5/subset B, 1, 3/{subset A, label=$b$, replaced
      labels=orange}, 4/subset B, 6/subset B}{2/3/subset A, 4/6/subset B,
      5/6/subset B}
```
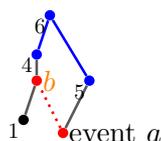

Note that `label=event $a$` and `label=$b$` adds (replaced) labels a and b, respectively, but their style is not affected by `subset A`. To change the style of the labels, we have to use the `replaced labels` key, see also Sec. 4.

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8 Externalisation and standalone causets

For any document that contains many (large) causet diagrams or the same causets that are used repeatedly, the compilation time can be reduced by moving the graphics to external files using the externalisation library of TikZ or `standalone` files.

The externalisation feature of TikZ provides the tools to automatically externalise all causet diagrams and only recompile them when they are changed in the document. To enable this feature, use the `external` option when loading the package and make sure that the document is compiled with the flag

```
1  --shell-escape
```

The `external` option loads the `external` library, sets the `prefix` to add "causets/" before each external file, and calls `\causetsNameExternaltrue` to use the manually specified names for external file names. Any causet can be manually named by providing the name key-value pair `name=⟨value⟩` in the optional parameter of a causet command. As an example, the inline code

```
1  \pcauset[name=Vee]{1,3,2}
```

yields the causet $\bigvee$ and creates an external pdf file with the name `causets/Vee.pdf` relative to the main tex file.

I recommend to always name causets, even if externalisation is not used. In this way, it is easier to read the source code and switch to externalising at a later point more easily. Note that the name of a causet does not have to be unique (especially if it is used repeatedly throughout the document), but you may give different names to causets with different style options to avoid unwanted overwrites. For a repeated causet in the document, either the same code is repeated, or the `\causetfile` command may be used, for example, `\pcauset[name=N]{2,4,1,3}` can be repeated with `\causetfile{N}` to print the N causet $\bigwedge\!\!\bigvee$. The macro `\causetfile[#1]{#2}` also accepts one optional argument `[#1]` and expands to

```
1  \causets@Padding%
2  \includegraphics[#1]{\causets@NamePrefix#2}%
3  \causets@Padding
```

To redefine the prefix stored in `\causets@NamePrefix` and also set the `external` library prefix to the same value, use `\tikzcausetsset{set prefix=causets/}`. However, to set `\causets@NamePrefix` only for `causets`, use `name prefix=⟨value⟩`.

If you are already loading the `external` library for other uses, you do not want to use the `external` option, but add the following to the document's preamble:

```
1  \usetikzlibrary{external}
2  ...
3  \usepackage{causets}
4  \tikzcausetsset{set prefix=causets/, name external=true}
```

For externalising only those causets that are named manually, set

```
1 \tikzset{external/only named=true}
```

For switching externalisation on/off all together, the library provides the macros

```
1 \tikzexternaldisable
2 \tikzexternalenable
```

Please find further information on the `external` library in the TikZ manual.

Graphics may also be moved to standalone tex files manually and included with `\causetfile` or `\includegraphics`. For example, create a file named `M.tex` in the directory that was set to `name prefix` (by default, this is "`./`") with the following code:

```
1 \documentclass[tikz]{standalone}
2 \usepackage{causets}
3 \begin{document}
4 \pcauset{4,2,5,1,3}
5 \end{document}
```

The compiled version ⅄⅄ is then included with the macro `\causetfile{M}`.

More examples on larger standalone graphics are given in Sec. 10.

9 Style changes in detail

In the optional argument of the causet macros, you may use other TikZ options to change the output of the macros, for example:

```
1 \usepackage{causets}
2 ... \begin{document} ...
3 A labeled causet with red-framed semi-transparent labels,
    \pcausetL[labels={draw=red, fill=white, opacity=0.8, inner sep=1pt,
    scale=1.5}]{2,4,5,1,3}.
```

A labeled causet with red-framed semi-transparent labels, .

In the same way, it is possible to modify the existing styles of all parts of the diagrams, which means that the default drawing styles are extended by the options passed as value to these keys:

1. `region=⟨value⟩` shows the permutation (`show permutation`) and appends to the style of the permutation frame,
2. `grid=⟨value⟩` shows the permutation (`show permutation`) and appends to the style of the permutation grid,
3. `tiles=⟨value⟩` shows the permutation (`show permutation`) and appends to the style of the occupied permutation tiles,
4. `events=⟨value⟩` appends to the style of the event nodes (graph vertices),
5. `links=⟨value⟩` shows the links (`show links`) and appends to the style of the links (main graph edges),
6. `link starts=⟨value⟩` sets the starting line tip of each link,
7. `link ends=⟨value⟩` sets the ending line tip of each link,

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8. `link pauses=⟨value⟩` sets the ending line tip of each link at an interruption,

9. `link resumes=⟨value⟩` sets the starting line tip of each link at an interruption,

10. `cap links` sets the starting and ending line tip of each link to a (reversed) cap,

11. `arrow links` sets the ending line tip of each link to a rounded, stealth arrow,

12. `spatial links=⟨value⟩` shows the spatial links (`show spatial links`) and appends to the style of the spatial links (secondary graph edges),

13. `all labels=⟨value⟩` appends to the general style of all text labels (applies to the following three label types),

14. `labels=⟨value⟩` shows the event labels (`show labels`) and appends to the style that applies only to the event labels,

15. `ulabels=⟨value⟩` shows the u labels (`show ulabels`) and appends to the style that applies only to the u labels, and

16. `vlabels=⟨value⟩` shows the v labels (`show vlabels`) and appends to the style that applies only to the v labels.

In order to manually define the full style of the parts in the diagram – not extending, but *overwriting all* drawing options – set the style of the following keys:

1. `every causet` for the general style of the causet diagram (like `baseline`, see below),
2. `every tile` for the occupied permutation tiles,
3. `every event` for the event nodes (graph vertices),
4. `every grid` for the permutation grid,
5. `every region` for the permutation region,
6. `every label base` for all text labels,
7. `every ulabel` for the permutation u labels,
8. `every vlabel` for the permutation v labels,
9. `every link` for the links (graph edges),
10. `every spatial link` for the spacelike separations, and
11. `every label` for the event labels.

With the macro `\tikzcausetsset{...}`, any of these styles can also be set in the preamble of the document. So, for example,

```
1 \usepackage{causets}
2 \tikzcausetsset{events={rectangle}, links={-Triangle Cap[]}}
3 ... \begin{document} ...
4 \pcauset[large]{2,4,5,1,3}
```

will generate all causets with square nodes as events and triangle caps on the future (upper) link ends, like ⬛╱╲⬛╲⬛.

By default, causets are placed such that a pre-defined coordinate point falls on the text baseline, `baseline=(baseline point)`. The baseline point is `tile size`$/\sqrt{2}$ below the centre of the diagram. You may adjust the baseline as for any TikZ graphic, for example,

```
1 \pcauset[baseline=2mm]{2,4,5,1,3}
```

places the diagram centre 2mm below the text base, .

10   Using this package in TikZ pictures

Since the package is TikZ based, using a causet within a TikZ picture is simple. Each of the three main commands is based on a draw command in TikZ that has the same name but starts with `\draw....` The `\draw` macros, however, do not support an optional argument (since this can be specified directly in a TikZ picture) and they also do not have any short hand macros ending in P, L or X. The `\draw...` do not add extra padding (`\causets@Padding`) on the left and right of the graphic either.

The two following examples are given by external (standalone) TikZ graphics and included with `\includegraphics{...}`. More details on using causets in standalone files are given in Sec. 8.

Note that the output of the `\draw` macros has the point (0,0) in the centre, so in order to shift it, we may enclose it in a TikZ scope, for example:

```
1  \documentclass[tikz]{standalone}
2  \usepackage[permutation,larger,neon]{causets}
3  \usetikzlibrary{fit,shapes.geometric}
4  \begin{document}
5  \begin{tikzpicture}
6    % Create causet, shifted in x- and y-direction:
7    \begin{scope}[xshift=3cm, yshift=-1cm]
8      \tikzcausetsset{offset=-2}
9      \drawpcauset{5,2,7,3,6,1,4}
10     \draw[red, double] (0, 0) circle[radius=0.1];
11   \end{scope}
12   % Mark the maximal elements (future infinity):
13   \node[draw=blue, inner sep=1pt, thick, ellipse, fit=(E2) (E4) (E5)]
       (Finf) {};
14   % Add labels on top:
15   \node[draw=blue, right, align=right] (FinfLabel) at (4.8, -0.5)
       {maximal events / \\ (future infinity)};
16   \node[draw=red] (centerLabel) at (0.5, -0.5) {centre};
17   \node[draw=green!50!black] (myEventLabel) at (5.5, -2.5) {event (E0)};
18   % Draw arrows from the labels to the references:
19   \draw[ultra thick, blue, ->] (FinfLabel) -- (Finf);
20   \draw[ultra thick, red, ->] (centerLabel) -- (3, -1);  % centre was
       shifted by the scope
21   \draw[ultra thick, green!50!black, ->] (myEventLabel) -- (E0);
22 \end{tikzpicture}
23 \end{document}
```
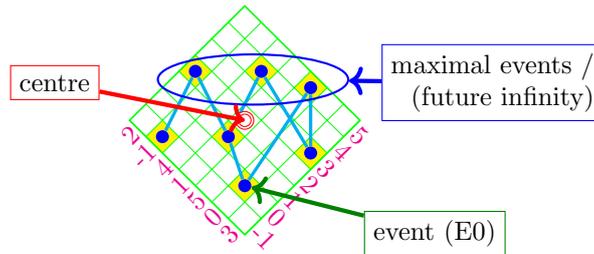
Note that this example also shows how to use the event nodes that are generated by the `\drawpcauset` macro. The green arrow points to the node of the second causet event (`E5`). The causet events, event labels, u-axis labels, and v-axis lables are the nodes (`Ei`), (`ELi`), (`EULi`), and (`EVLi`), respectively, where $i \in [-1, 5]$.

It is also possible to use multiple causets in more complicated graphics, like a tree for example, but note that whenever a causet is actually a text of a node within a TikZ picture (like in the following), we need to use the `\pcauset` (`\rcauset` or `\causet`) commands without the `draw` prefix again:

```
1  \documentclass[tikz]{standalone}
2  \usepackage[black]{causets}
3  \usetikzlibrary{fit,shapes.geometric}
4  % Set colours and style:
5  \colorlet{future colour}{green!50!black}
6  \colorlet{past colour}{yellow!50!orange!80!black}
7  \colorlet{anywhere colour}{red!80!black}
8  \tikzset{prob arrow/.style={line width=1.5pt}}
9  % Define some helper functions:
10 \def\defCevents#1#2#3{\xdef\Ea{#1}\xdef\Eb{#2}\xdef\Ec{#3}}
11 \def\defDevents#1#2#3#4{\expandafter\defCevents#1#2#3\xdef\Ed{#4}}
12 \newcommand*{\semiopaque}[1]{% Adds a transparency group.
13 \begin{scope}[transparency group, opacity=0.5]
14   #1
15 \end{scope}
16 }
17 \newcommand*{\drawprobarrow}[4][]{% Draws a (probability) arrow.
18 \draw[prob arrow] (#2) -- node[sloped, midway, below, #1] {$#3$} (#4);
19 }
20 \newcommand{\drawlegendsymbol}{% Draws the symbol for a legend entry.
21 \draw[prob arrow] (0,0) -- +(10pt, 6pt);
22 }
23 \newcommand{\legendsymbol}[1]{% Creates the legend symbol.
24 \begin{tikzpicture}
25   \ifnum#1=0
26     \drawlegendsymbol
27   \else
28     \semiopaque{\drawlegendsymbol}
29   \fi
30 \end{tikzpicture}
31 }
```

```latex
32  \begin{document}
33  \begin{tikzpicture}[-stealth]
34    \def\ystep{2.0cm}
35    \def\xstep{0.52cm}
36    % Create nodes of all causets with 1 to 4 events:
37    \begin{scope}[nodes={draw, thin, circle, minimum size=1.02cm}]
38      \node (C1) at ( 0,  0) {\pcauset{1}};
39      \node (C12) at (-4*\xstep,  1*\ystep) {\pcauset{1,2}};
40      \node (C21) at ( 4*\xstep,  1*\ystep) {\pcauset{2,1}};
41      \foreach \perm [count=\i from 0] in {123, 132, 312, 213, 321}{%
42        \expandafter\defCevents\perm
43        \node (C\perm) at (4*\i*\xstep-8*\xstep, 2*\ystep)
      {\pcauset{\Ea,\Eb,\Ec}};
44        }
45      \foreach \perm [count=\i from 0] in {1234, 1243, 1423, 1324, 1432,
      4123, 4132, 3124, 3412, 3142, 4312, 2134, 2143, 4213, 3214, 4321}{%
46        \expandafter\defDevents\perm
47        \node (C\perm) at (2*\i*\xstep-15*\xstep, 3.75*\ystep)
      {\pcauset{\Ea,\Eb,\Ec,\Ed}};
48        }
49    \end{scope}
50    % Add graph edges for expandings ...
51    % ... to the past (additionally):
52    \begin{scope}[past colour]
53      \foreach \perma/\permb in {12/213, 123/3124, 123/2134, 132/1243,
      132/3142, 132/2143, 312/1423, 312/4213, 213/1324, 213/3124,
      213/3214, 321/1432, 321/4132, 21/132}
54        \semiopaque{\drawprobarrow{C\perma}{}{C\permb}};
55    \end{scope}
56    % ... to anywhere (additionally, further):
57    \begin{scope}[anywhere colour]
58      \semiopaque{\drawprobarrow{C123}{}{C1324}};
59    \end{scope}
60    % ... to the future:
61    \begin{scope}[prob arrow, future colour]
62      \drawprobarrow{C1}{p_{12}}{C12}
63      \drawprobarrow{C12}{p_{123}}{C123}
64      \drawprobarrow{C123}{p_{1234}}{C1234}
65      \drawprobarrow[near end]{C123}{p_{1243}}{C1243}
66      \drawprobarrow[near end]{C123}{p_{1423}}{C1423}
67      \drawprobarrow[near start, above]{C123}{p_{4123}}{C4123}
68      \drawprobarrow[above]{C12}{p_{132}}{C132}
69      \drawprobarrow[near end]{C132}{p_{1342}}{C1423}
70      \drawprobarrow[near end]{C132}{p_{1324}}{C1324}
71      \drawprobarrow[midway, above]{C132}{p_{1432}}{C1432}
72      \drawprobarrow[near start]{C132}{p_{4132}}{C4132}
73      \drawprobarrow[near start, above]{C12}{p_{312}}{C312}
74      \drawprobarrow[near end]{C312}{p_{3124}}{C3124}
```

```
75      \drawprobarrow[near end]{C312}{p_{3412}}{C3412}
76      \drawprobarrow[near end]{C312}{p_{3142}}{C3142}
77      \drawprobarrow[near start]{C312}{p_{4312}}{C4312}
78      \drawprobarrow{C1}{p_{21}}{C21}
79      \drawprobarrow[near start, above]{C21}{p_{231}}{C312}
80      \drawprobarrow[near start]{C312}{p_{2341}}{C4123}
81      \drawprobarrow[near end]{C312}{p_{2431}}{C4132}
82      \drawprobarrow{C21}{p_{213}}{C213}
83      \drawprobarrow[above]{C213}{p_{2134}}{C2134}
84      \drawprobarrow[near start]{C213}{p_{2413}}{C3142}
85      \drawprobarrow[near end]{C213}{p_{2143}}{C2143}
86      \drawprobarrow[near end]{C213}{p_{4213}}{C4213}
87      \drawprobarrow{C21}{p_{321}}{C321}
88      \drawprobarrow[near start, above]{C321}{p_{3421}}{C4312}
89      \drawprobarrow[near end]{C321}{p_{3241}}{C4213}
90      \drawprobarrow[near end]{C321}{p_{3214}}{C3214}
91      \drawprobarrow{C321}{p_{4321}}{C4321}
92    \end{scope}
93    % Add legend:
94    \node[above left, align=left, draw, inner sep=3ex, fill=gray!05]
95      at (15*\xstep, 0)
96      {Expanding towards\\
97       \bfseries\color{future colour}\legendsymbol{0}\ the future\\
98       \bfseries\color{past colour}\legendsymbol{1}\ + the past\\
99       \bfseries\color{anywhere colour}\legendsymbol{1}\ + anywhere};
100 \end{tikzpicture}
101 \end{document}
```
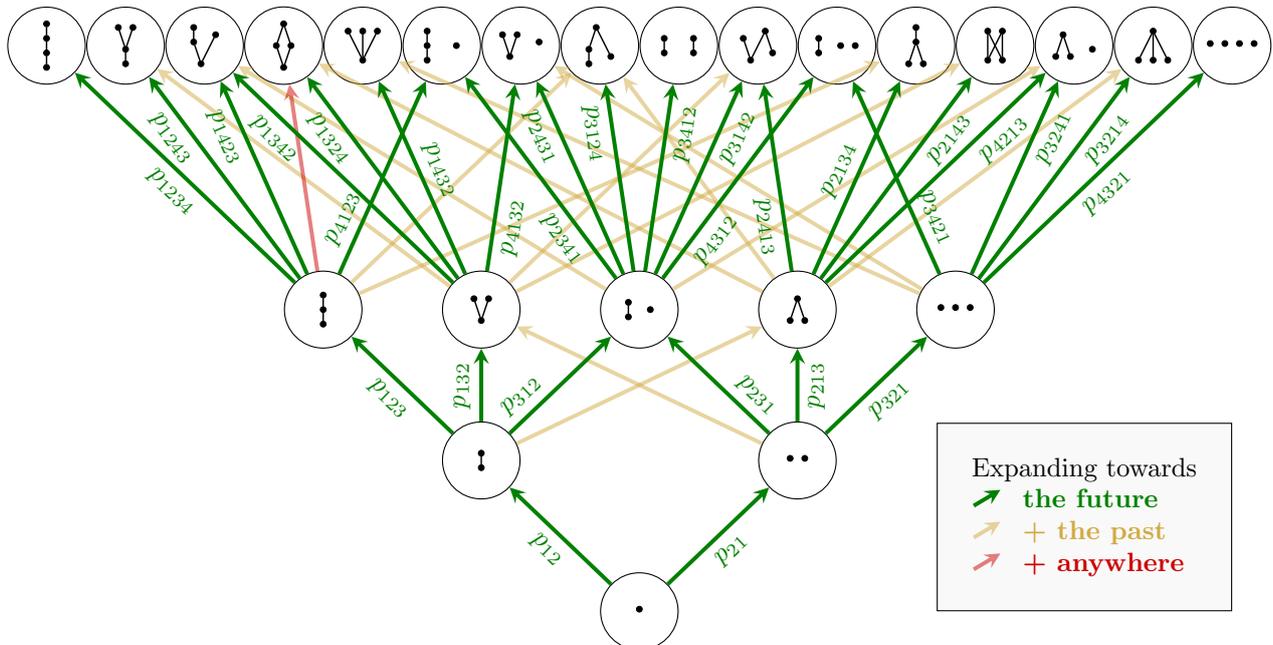

More examples can be found at

https://github.com/c-minz/LaTeX-causets/tree/main/examples

11   Release notes

v1.4, 2023-03-06

New features

- Second values in the sequence of the first argument: now modifies the style of individual events, the events are replaced with the key `event=⟨value⟩`, and the labels are set with `label=⟨value⟩`.
- Third values in the sequence of the second argument of `\causet` and `\rcauset`: to modify the style of individual links.
- Broken links for restyled links of `\rcauset`: restyled links can also be drawn between events that would have not been automatically linked, hence these links might now be broken up like the links of `\causet`.
- New key `name=⟨value⟩`: to define a name of a causet for easier readability of the tex source code (and to be used as external file name when the corresponding package option has been activated).
- New option `external`: to load the `external` library of TikZ for automatic externalisation of all diagrams.
- New key `set prefix=⟨value⟩`: to pass a file name prefix that is combined with a causet name for the external files when using `external`.
- New key `name external=⟨bool⟩`: to activate/deactivate using the causet names for external files.
- Main commands: padded by a thin space (adjustable with `padding=⟨value⟩`) for a better spacing in text and math mode.

Modifications and fixes

- Third values in the sequence of the first argument: no longer supported.
- Spatial links: are now drawn before (so on a lower layer than) links.
- Style `arrow links`: the arrow heads are now filled.
- Macro `\causetspath`: has been replaced by `name prefix=⟨value⟩`.
- Short-hand macro `\causetCrown`: changed to take a single argument (and the optional style argument) and returns the same output as `\causetClosedFence` used to, which has been removed.

v1.3.1, 2022-07-27

Modifications and fixes

- Automatic event labels: now correctly printed when using `offset=⟨value⟩`.

v1.3, 2022-05-23

New features

- Integer sequences in the arguments: use a permutation of an interval $[a, b]$, where a and b do no longer have to be natural numbers but can be any integers ($a \leq b$)

including 0 and negative numbers. These numbers name and label the events of the causet if the respective options are chosen.

- Second values in the sequence of the first argument: to change the name and label of events.
- New key `offset=`⟨*value*⟩ (0 by default): to define an integer shift for all events before they are named and labeled.
- New style `text font=`⟨*value*⟩ (additionally to `text scale=`⟨*value*⟩): to adjust the font style of text (labels), for example, to set the font size to 20pt (and a baseline skip of 24pt), use `text font=\fontsize{20}{24}\selectfont`

Modifications and fixes

- Size options: all sizes are now scaled with respect to the normal size.
- Event style: line thickness of the event outline has been corrected.
- Size macros removed: `\causetRegionLine`, `\causetGridLine`, `\causetEventSize`, `\causetLinkWidth`, and `\causetBrokenLinkGap`. The respective values are now stored in PGF keys and are measured with respect to the size unit (stored in `\causetTileSize`).
- All package options: reduced to single words.
- Permutation for `\*causetL...`: now deactivated (if activated previously), by default.
- Labels for `\*causetP...`: now deactivated (if activated previously), by default.

v1.2, 2022-02-07

New features

- New style `at=`⟨*value*⟩: to specify the relative point of events (`at=center`⟨*value*⟩, by default) and position relative to this point using TikZ keys `left=`⟨*value*⟩, `right=`⟨*value*⟩, etc.
- New styles `cap links` and `arrow links`: add different arrow tips to the start and end of links.
- New styles `link starts=`⟨*value*⟩ and `link ends=`⟨*value*⟩: to modify arrow tips to the start and end of links.
- Broken links for `\causet`: links crossing over unlinked events are broken up now.
- New styles `link pauses=`⟨*value*⟩ and `link resumes=`⟨*value*⟩: to set the link ending style (arrows) at breaking points.
- Spatial links for `\rcauset`: spatial links for spacelike separated events are now supported by `\rcauset`, too.

Modifications and fixes

- Colour option `blue`: blue colours are now different shades of blue, gray and black only.

v1.1, 2020-12-15

New features

- All colours, size and style options have been moved to PGF keys in the path

```
1 /tikz/causets/
```

v1.0, 2020-11-08

- First release of the package `causets` on CTAN, https://ctan.org/pkg/causets, including all the main commands.

All previous versions of the package can be downloaded at
https://github.com/c-minz/LaTeX-causets/tree/main/previous_versions

12   Bug reports and package requests

To see reported problems, go to https://github.com/c-minz/LaTeX-causets/issues
If you have a problem when using the package or you would like to have another feature to be implemented, please write a message to
christoph(dot)minz(AT)gmail(dot)com
For reporting a bug, I much appreciated if you do the following:

- Check the issues reported previously if your problem is already listed.
- Describe your problem including the errors and warning messages, information about the drivers and programs versions.
- Provide a minimal, working test file (only a single TeX file) that demonstrates the problem.